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Optimal Use of TDOA Geo-Location Techniques Within the Mountainous Terrain of Turkey

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13. ABSTRACT

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These techniques are important for military applications. Location finding is a part of electronic warfare support, which is one of the main braches of electronic warfare. Because these techniques are platform independent, they can be used with any system or platform, such as UAVs, manned aircraft, ground locations, etc. In Turkey it is vitally important for the army conducting search and destroy operations against terrorists to locate emitters associated with these terrorists.

The simulation developed in this thesis provides a better understanding of the accuracy of TDOA based geolocation systems. Combinations of receivers and techniques are explored to determine the optimal solutions. The factors of noise and distance have a linear effect on accuracy. The best combination of receivers is determined with consideration to using a combination of fixed and airborne platforms. The best distribution for highest accuracy is determined and discussed.

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OPTIMAL USE OF TDOA GEO-LOCATION TECHNIQUE DUE TO MOUNTAINOUS TERRAIN OF TURKEY

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Captain, Turkish Army
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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

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LIST OF ACRONYMS AND ABBREVIATIONS

2D	Two-Dimensional
3D	Three-Dimensional
AOA	Angle of Arrival
CW	Continuous Wave
DD	Differential Doppler
DF	Direction Finding
EA	Electronic Attack
EM	Electromagnetic
EMS	Electromagnetic Spectrum
EP	Electronic Protection
ES	Electronic Warfare Support
EW	Electronic Warfare
FDOA	Frequency Difference of Arrival
FM	Frequency Modulation
GPS	Global Positioning System
HF	High Frequency
IA	Information Assurance
IF	Intermediate Frequency
IO	Information Operations
JP	Joint Publication
LOB	Line of Bearing
LOP	Line of Position

PF   Position Fix

PKK   Partiya Karker Kurdistan (Kurdistan Worker's Party)

RF   Radio Frequency

SHF   Super High Frequency

SIGINT   Signal Intelligence

SNR   Signal to Noise Ratio

TDOA   Time Difference of Arrival

TOA   Time of Arrival

UAV   Unmanned Air Vehicle

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DISCLAIMER

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Turkish Republic, the Turkish Armed Forces, the Turkish Land Forces, the Turkish Naval Forces or the Turkish Air Force.

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I.   INTRODUCTION

A.   AREA OF RESEARCH

Electronic warfare (EW) plays a dominant role in today's world of technological warfare. It has been considered to be a force multiplier for decades. Nations who understand the importance of the EW have always used it and benefited. We as the soldiers cannot imagine a battlefield without EW and its components. "EW is one of the five core capabilities" (JP 3-13-1, 2007). Joint Publication (JP) 3-13-1 Electronic Warfare is U.S. joint doctrine for EW and provides joint doctrine for every part of EW, ranging from planning and preparation to execution for military operations. This publication is an important source for anyone wanting to define what EW is and what components compose it. We cannot win a war or even a battle without the use of EW.

EW includes three major branches: Electronic Attack (EA), Electronic Protection (EP) and Electronic Warfare Support (ES). "EA involves the use of EM energy, directed energy, or anti-radiation weapons to attack personnel, facilities, or equipment with the intent of degrading, neutralizing, or destroying enemy combat capability and is considered a form of fire. EP involves actions taken to protect personnel, facilities, and equipment from any effects of friendly or enemy use of the electromagnetic spectrum that degrade, neutralize, or destroy friendly combat capability. ES is the subdivision of EW involving actions tasked by, or under direct control of, an operational commander to search for, intercept, identify, and locate or localize sources of intentional and unintentional radiated EM energy for the purpose of immediate threat recognition, targeting, planning, and conduct of future operations" (JP 3-13-1, 2007).

ES includes direction finding of enemy RF signals. Direction finding can also be considered as finding the enemy location by analyzing the signal or signals that are transmitted by the emitter. This is also called emitter location or, if the emitter is on the earth's surface, emitter geo-location.

It is important for both the military and civilians to be able to find the location of RF emitters on the surface of the earth under extreme conditions, such as in war for the military and in the case of accidents or disasters for civilian response. Military forces want to know the where the enemy is. Determining the location of enemy forces has always been important to combat, and various methods have been studied and developed through the ages. The easiest and most accurate way to do this historically has been to send a man forward to locate the enemy. Time and available human resources limit this. There are inherent risks, such as having the soldiers killed and being recognized by the enemy forces if they capture the reconnaissance team. Because of these limitations people have tried to use technology as much as possible in the modern era.

It is important to know the position of the enemy without letting them know that you are trying to locate them. This can be done with passive location finding techniques. In contrast, people have been using active location finding techniques, such as radar, for decades. When active location finding techniques are used they let the enemy know that they are being identified or located because active location finding techniques uses detectable RF signals.

To reduce the chance that the enemy will discover that they are under scrutiny, scientists and engineers have developed techniques for passive location finding. These techniques depend upon the interception of enemy transmissions within the EM spectrum, rather than emanating friendly EM transmissions. Some of the more common of these techniques are

- TDOA (Time Difference of Arrival)

- FDOA (Frequency Difference of Arrival)

- AOA (Angle of Arrival)

- TOA (Time of Arrival)

Among these techniques, TDOA is currently the most commonly studied technique. This technique is platform independent. TDOA is most easily applied to pulsed signals. Since all signals of interest are not pulsed signals, some techniques have to be implemented for continuous wave (CW) signals. Pulsed signals have distinctive features that can be recognized in the waveform. CW signals do not have the same distinctive features and can change over time. The signal depends on the message that is carried over the carrier. Since the message over the carrier is random, it is difficult to determine the beginning and the end of the signal.

Radars typically use pulsed signals, and communication systems typically use various modulations of CW signals. Ground forces are normally more interested in communication signals than radar; military forces need to coordinate their needs with scientists to find an appropriate geolocation technique for the CW signals associated with these communications.

The FDOA technique requires a long-duration signal to be able to determine direction to a sufficient accuracy for position determination. Conversely, the TDOA technique does not need this long duration. Instead of requiring a long duration signal, very short duration signals can be used to determine the location if all the necessary receivers receive the signal. Proper coordination of the signals between the receivers and the main node that compute the position, and synchronization among the receivers, are necessary for accurate emitter position estimation.

For ground forces, passive location finding becomes important for locating enemy combatants or terrorists in mountainous areas, such as those found in the southeast part of Turkey. Turkish land forces have been conducting search and destroy operations for over 30 years in this region. As a part of these operations, Turkish Land Forces typically first attempt to locate the terrorists who hide within the cave infrastructure common to the area and then destroy them. Terrorists in the region are known to move as small groups and to use frequency modulation (FM) based communication devices. If friendly forces can use a passive

technique to locate them this would make the search process easier. Depending upon the operation, an accuracy of 100–200 meters is sufficient. For this purpose, unmanned air vehicles (UAVs) can be used. Since typical insurgent FM communications are not long-duration or long distance, troops should have mobile direction finding (DF) devices with them or nearby.

High clutter brings out another problem. When troops use DF devices in mountainous areas such as southeast Turkey, devices might receive signals more than once due to the reflection of the signal from the clutter. This makes the problem more complicated and more difficult, especially when troops want to locate the enemy emitter in three dimensions.

1. Tactical Situation

This thesis is concerned with a tactical situation in Turkey.

Turkish armed forces has been planning and conducting military operations against the PKK (Partiya Karker Kurdistan (Kurdistan Worker's Party)). The PKK, whose main purpose is to weaken the ruling government in Turkey, is classified as a terrorist organization. Most nations of the world accept this classification.

These operations are conducted mostly in the southeast region of the Turkey. That part of Turkey is very mountainous and includes many caves. Terrorists hide in the caves and come out whenever they want to attack. Operating within mountainous terrain gives them an advantage in hiding from government forces and helps them to move more discretely. If military forces try to follow them, especially at night, they escape easily because they know the terrain better than the military forces. Darkness covers them and mountains hinder the movement of the larger military forces more than the terrorists.

These terrorists mostly come from Iraq, Iran and some other neighbor countries where they receive their training. They learn how to fight and use mines, explosives and other weapons and equipment. They cross the border in

small groups at night. They travel to their destinations on foot, sometimes taking from one to two weeks.

The terrorists typically use hand-held radio devices for communication between groups and the main camp if they are close to it. Some high-ranking terrorists use cell phones or satellite phones if they can, but GSM companies do not cover most of the mountainous areas where they operate. The devices used between groups normally use FM modulation. They try to use these devices as little as possible to avoid detection and location. Often the duration of these communications number in the seconds or at best tens of seconds.

FDOA cannot be used because of its long duration signal requirement. TDOA therefore, since it does not have the same requirement, might be the best solution.

Turkish forces have been using some technology and intelligence support to find the terrorists, but they still manage to escape using the methods discussed above.

These terrorists must be found and destroyed. A better method to find them is to use some sort of geo-location system targeted against their communications signals. Even though they try to use FM as little as possible, TDOA is a very good technique for this purpose. If land forces can use a tactic based on TDOA they will be more successful in finding, locating, and destroying the groups of terrorists.

2.   TDOA

Time Difference of Arrival is one of the basic DF techniques used to locate RF emitters for CW signals. "TDOA takes advantage of the fact that a transmitted signal will arrive at different sensors at different times." (Batson, McEachen, & Tummala, 2012) "A number of spatially separated sensors capture the emitted signal and the time differences of arrival (TDOAs) at the sensors are determined.

Using the TDOA's, emitter location relative to the sensor can be calculated." (Chan & Ho, 1994)

B.   MAJOR RESEARCH QUESTIONS

This thesis will explain the following subjects.

- What are EW and its components?

- What is the role of DF in EW?

This study will answer the following questions:

- What are the advantages and disadvantages of using TDOA?

- How effectively can ground troops use TDOA DF techniques in a high clutter area with noise?

- How can we simulate TDOA technique?

- What are the optimal uses of the TDOA technique?

C.   IMPORTANCE AND BENEFITS OF THE STUDY

This study can be used as a guide for developing a geolocation system based on TDOA. It will also identify the benefits of using a TDOA-based geolocation system for ground forces in high clutter terrain. It will clarify the major EW components and their usage as an introduction to discussions of TDOA.

D.   ORGANIZATION OF THE THESIS

This thesis is composed of five chapters. Chapter I introduces the area of research, major research questions, importance and benefits of the study and the organization of the thesis.

Chapter II presents information about EW fundamentals. EW and its components are examined. The relationship between EW and IO is explained. The importance of EW is explained.

Chapter III explains DF techniques such as directional antenna, phase interferometry, TDOA, and FDOA. TDOA, as a part of ES, is explained briefly. The details of the TDOA technique and its usage in the field are examined. Capabilities of TDOA are described. The developed TDOA-based location system for ground forces in a high clutter area are explained from Ezzat's paper. The least square estimation is explained and the required recursive least square estimation approach is examined in detail. Sources of error are explained for DF and TDOA.

Chapter IV explains the developed simulation for the system. Assumptions and restrictions for the simulation are listed. After running the simulation, five types of analyses are done to see their effects on the accuracy of the proposed system and the simulation.

Chapter V is the conclusion. The results obtained from the system simulation are explained. Recommendations for future works are given.

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II.   ELECTRONIC WARFARE

In this chapter, information operations and its relationship to electronic warfare are defined. Then electronic warfare and its components explained. Since the topic of the thesis is direction finding, and because direction finding is a subdivision of electronic warfare support (ES), ES is explained in more detail. These definitions are made according to Frater and Ryan's book on EW, with secondary reference using U.S. EW doctrine as discussed in Joint Publication 3.13.1.

Another classification of EW components is based on their activities, which can be defined as either active or passive. Active/passive classification is discussed.

A.   INFORMATION OPERATIONS

Information has a vital importance for any nation and for its existence. Every nation around the world tries to obtain information, use it as much as possible for its own good, and prevent the enemy from taking advantage of it. Information operations (IO) is the effort done for this purpose, by human or by machine. IO incorporates the actions taken to preserve the integrity of one's own information system from exploitation, corruption, or disruption, while at the same time exploiting, corrupting and destroying the adversary's information system (Adamy, 2004).

Using U.S. doctrine as a reference, IO coordinates and synchronizes five core capabilities to help the commander to reach his/her purpose in the battlefield. These capabilities are psychological operations, military deception, operations security, electronic warfare and computer network operations (JP 3.13, 2006). In addition to the core capabilities doctrine defines supporting capabilities, which are information assurance (IA), physical security, physical attack, counterintelligence, and combat camera, (JP 3-13, 2006) and related

capabilities of civil military operations, public affairs and defense support to public diplomacy (JP 3-13, 2006).

The world has gone to cellular and wireless. Since the Information Age produced a revolution in military operations, the electromagnetic spectrum is essential to the transmission and reception of information in the modern world, and therefore EW is a critical element of information operations. As communication and information systems become increasingly vital for military and civilian society, they can become targets in war for an enemy; therefore, they can play a significant role for offensive and defensive operations. The military has adopted communication and information systems and they have become essential for military operations. Commanders need information to be able to cope with the complexity of modern warfare. This reliance on information in turn makes them vulnerable to attack. There is an emphasis for commanders to attack adversary information and communication systems. Modern battlefields rely heavily on the use of the electromagnetic spectrum (EMS), whether for surveillance and target acquisition, passage of information, processing of information, or destruction of enemy forces (Frater, & Ryan, 2001). Figure 1 shows all the necessary capabilities for IO and their roles.

INFORMATION OPERATIONS INTEGRATION INTO JOINT OPERATIONS (NOTIONAL)

Core, Supporting, Related Information Activities	Activities	Audience/ Target	Objective	Information Quality	Primary Planning/ Integration Process	Who does it?
Electronic Warfare	Electronic Attack	Physical, Informational	Destroy, Disrupt, Delay	Usability	Joint Operation Planning and Execution System (JOPES)/ Targeting Process	Individuals, Governments, Militaries
	Electronic Protection	Physical	Protect the Use of Electro-magnetic Spectrum	Security	JOPES/Defense Planning	Individuals, Businesses, Governments, Militaries
	Electronic Warfare Support	Physical	Identify and Locate Threats	Usability	Joint Intelligence Preparation of the Battlespace(JIPB)/SIGINT Collection	Militaries
Computer Network Operations	Computer Network Attack	Physical, Informational	Destroy, Disrupt, Delay	Security	JIPB/JOPES/Targeting Process	Individuals, Governments, Militaries
	Computer Network Defense	Physical, Informational	Protect Computer Networks	Security	JOPES/J-6 Vulnerability Analysis	Individuals, Businesses, Governments, Militaries
	Computer Network Exploitation	Informational	Gain Information From and About Computers and Computer Networks	Security	JIPB/Targeting Process	Individuals, Governments, Militaries
Psychological Operations	Psychological Operations	Cognitive	Influence	Relevance	JOPES/Joint Operation Planning	Businesses, Governments, Militaries
Military Deception	Military Deception	Cognitive	Mislead	Accuracy	JOPES/Joint Operation Planning	Militaries
Operations Security	Operations Security	Cognitive	Deny	Security	JOPES/Joint Operation Planning	Businesses, Governments, Militaries
Supporting Capabilities	Information Assurance	Informational	Protect Information and Informatin Systems	Security	JOPES/J-6 Vulnerability Analysis	Businesses, Governments, Militaries
	Physical Security	Physical	Secure Information and Information Infrastructure	Usability	JOPES/Defense Planning	Businesses, Governments, Militaries
	Physical Attack	Physical	Destroy, Disrupt	Usability	JOPES/Joint Operation Planning	Governments, Militaries
	Counterintelligence	Cognitive	Mislead	Accuracy	JIPB/Human Intelligence Collection	Governments, Militaries
	Combat Camera	Physical	Inform/Document	Usability, Accuracy	JOPES/Joint Operation Planning	Governments, Militaries
Related Capabilities	Civil Military Operations	Cognitive	Influence	Accuracy	JOPES/Joint Operation Planning	Governments, Militaries
	Public Affairs	Cognitive	Inform	Accuracy	JOPES/Joint Operation Planning	Businesses, Governments, Militaries
	Public Diplomacy	Cognitive	Inform	Accuracy	Interagency Coordination	Governments

Figure 1.   Information Operations Integration (From: JP 3.13)

B.   ELECTRONIC WARFARE AND ITS COMPONENTS

Electronic Warfare is one of the five core capabilities of IO. EW dominates the EMS for the benefit of friendly forces. The EMS is shown in Figure 2 and Figure 3. EW can be defined as "… any military action involving the use of electromagnetic (EM) and directed energy to control the EM spectrum or to attack the adversary." (JP 3.13, 2006) The means to conduct EW targeting are typically technological in nature, and the immediate targets are normally technological, but the ultimate target is the commander's decision-making capability. If EW is successful, the friendly commander can make good decisions based on the information coming from technological devices; on the other hand, the adversary commander cannot make good decisions because he/she cannot access good and useable information as much as he/she needs to.
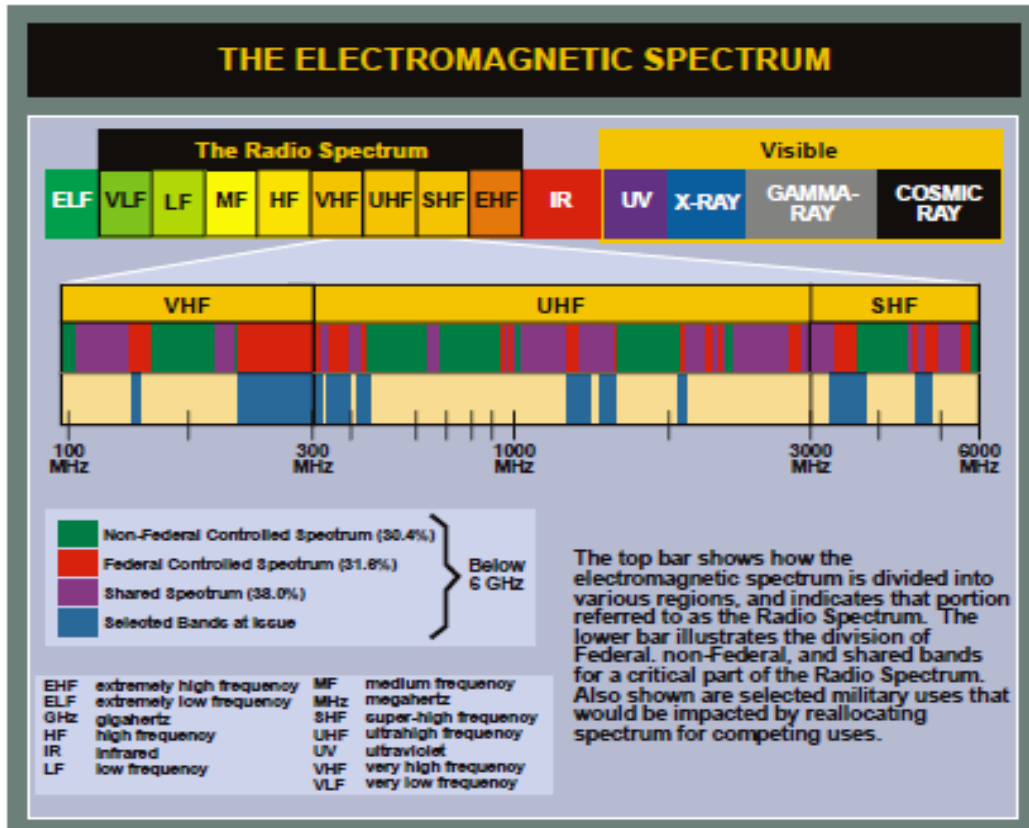
Figure 2.   Electromagnetic Spectrum (From JP 3.13.1)
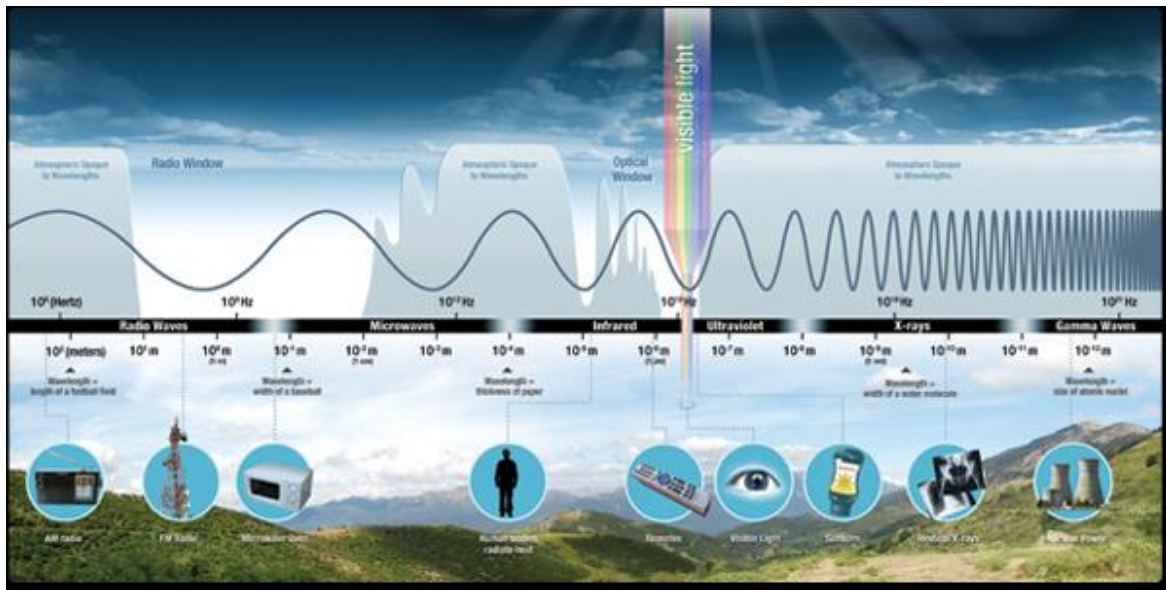
Figure 3.   Electromagnetic Spectrum (From: National Aeronautics and Space Administration, Science Mission Directorate)

All components of EW can be applied to all kind of operations. During peacetime, military forces try to use EW to detect potential adversary EMS usage and gather intelligence; during wartime, they use EW to protect their own EMS usage ability and prevent adversary usage.

EW can be divided into two parts: communications EW and non-communications EW. Communications EW is mostly concerned with communication sources that transmit in frequency bands between HF (3–30 MHz), VHF (30–300 MHz), UHF (300–3000 MHz), and SHF (3000 MHz to 30 GHz). Non-communications EW is mainly concerned with radar systems, of which some operate in the lower communications bands, but most are located in the higher microwave and millimeter wave frequencies. Additionally, research in directed energy weapons and technology is increasing in importance.

As shown in Figure 4, EW has three doctrinal subdivisions.
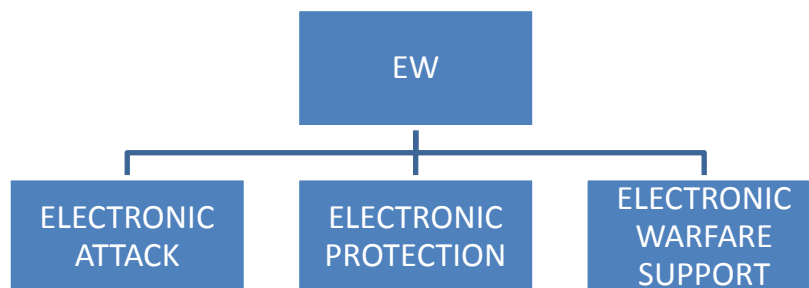
Figure 4.   Subdivisions of EW

1.   Electronic Attack

EA involves actions taken against personnel, equipment or facility to degrade their use of the EMS and combat abilities.

Subdivisions of EA are shown in Figure 5.
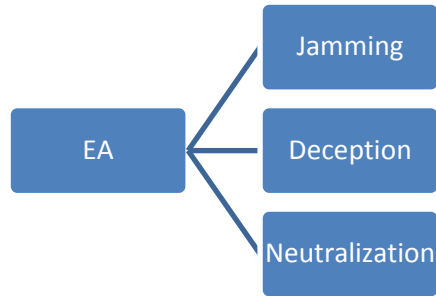
Figure 5.   Subdivisions of EA (From Frater & Ryan, 2001)

2.   Electronic Protection

EP involves actions taken to protect personnel, equipment and facilities from any effect of friendly or adversary EW activities that degrade, neutralize or destroy friendly combat capabilities.

Subdivisions of EP are shown in Figure 6.
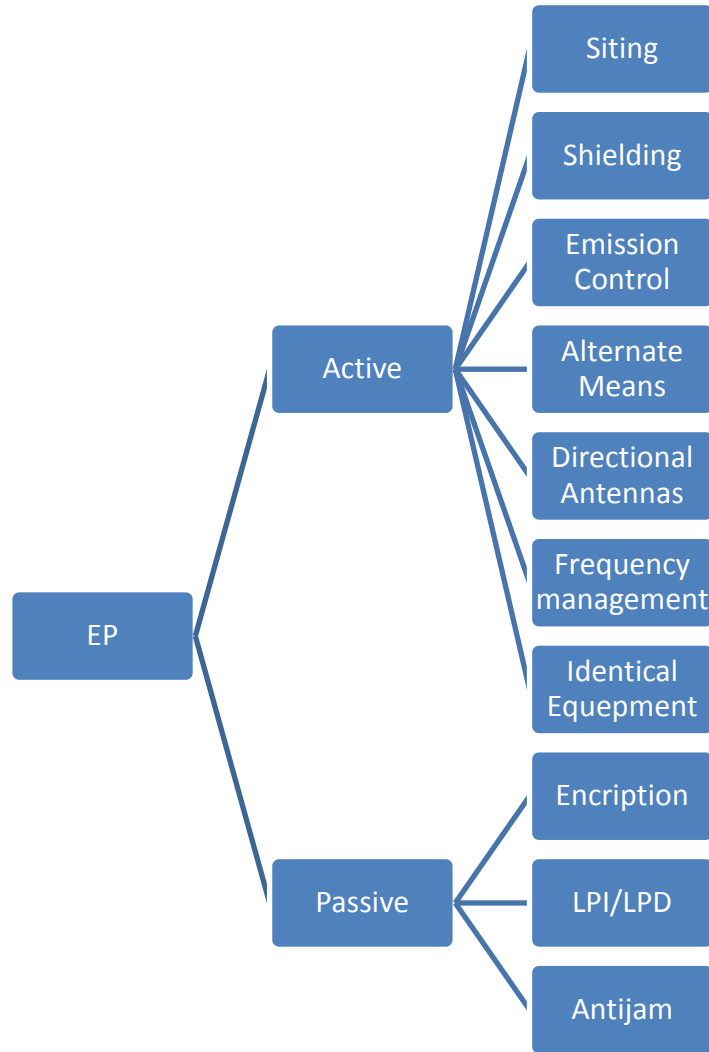
Figure 6.   Subdivisions of EP (From Frater & Ryan, 2001)

3.   Electronic Warfare Support

Since this thesis focuses on locating adversary emissions in the difficult terrain of eastern Turkey, our discussion of ES will be more extensive and relevant to the thesis than the other doctrinal subdivisions of EW. ES involves actions taken to identify, intercept and locate intentional or unintentional radiated electromagnetic energy. The purpose of the ES is target recognition. The main functions of ES are to produce intelligence, to produce steerage for EA and to cue surveillance and target acquisition resources. (Frater & Ryan, 2001)

Subdivisions of ES are shown in Figure 7.
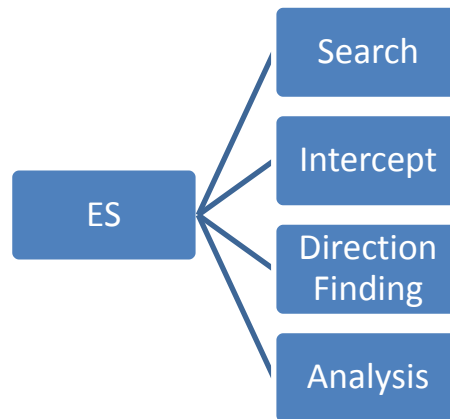
Figure 7.   Subdivisions of ES (From Frater & Ryan, 2001)

ES differs from, but is similar to Signals Intelligence (SIGINT). ES is used for immediate battlefield information and SIGINT is used for intelligence. ES supports near term operational applications and SIGINT supports long term applications. The combat information gathered by ES can be provided to intelligence resources in addition to being used operationally. Combat information does not normally require the type of deep analysis that SIGINT typically does.

Previously shown in Figure 7 were the subdivisions of ES, which we will now discuss in more detail.

Search: it is necessary to search for and identify the EM signal that the adversary uses in the EMS before it can be examined.

The search systems act in space, time, and frequency. They have to be close enough to the adversary's transmitting system to be able to detect the signal. They have to be actively searching or listening at the same time that the adversary system is transmitting. Finally, they must be listening to the same frequencies used by the transmitter. This frequency requirement is generally met with two compatible technology approaches, narrowband receivers and broadband receivers. Narrowband receivers can receive a single signal at a time and can scan a desired bandwidth sequentially in frequency; on the other hand, broadband receivers can monitor multiple channels at the same time.

Intercept: Once the signals of interest are identified through the search process they have to be analyzed during intercept based on their modulation, bandwidth, amplitude, frequency and other parameters. This process is also called monitoring.

Direction Finding (DF): The location of the transmitter is determined by information gathered during the search and intercept process. These locations are likely to be an approximation rather than an exact location. DF was historically based on triangulation where there had to be at least three receivers around the emitter.

DF systems historically employed special antennas, which defined the bearing towards the emitter. When the lines of bearing from each receiver are drawn on a map manually or automatically, the interception of the lines form a triangle. The smaller the triangle, the better the accuracy of the system. The emitter lies inside the triangle.

Analysis: Once the signals are examined they are analyzed to define the adversary's electronic warfare capabilities. The main purpose is to clarify the battlefield for the commander from the EMS perspective.

ES targets an adversary's EA, communication systems and electronic systems. A typical target is an adversary's communication systems, where the information gathered is used for operationally actionable intelligence and targeting purposes.

Because JP 3.13.1 is the shaping document of EW for the U.S. and many of its allies, it is a good idea to supplement Frater and Ryan's book with discussions from the joint publication. According to JP 3.13.1, EA has five subdivisions: Electromagnetic Jamming, Electromagnetic Deception, Directed Energy, Anti-radiation Missiles and Expendables (e.g., chaff, flares and active decoys). ES has three subdivisions: Threat Warning, Collection Supporting EW and Direction Finding. EP has three subdivisions: Spectrum Management, EM Hardening and Emission Control. Subdivisions of EW do not act alone, they

interact with each other. The interaction among the subdivisions of EW and their subdivisions are shown in Figure 8.
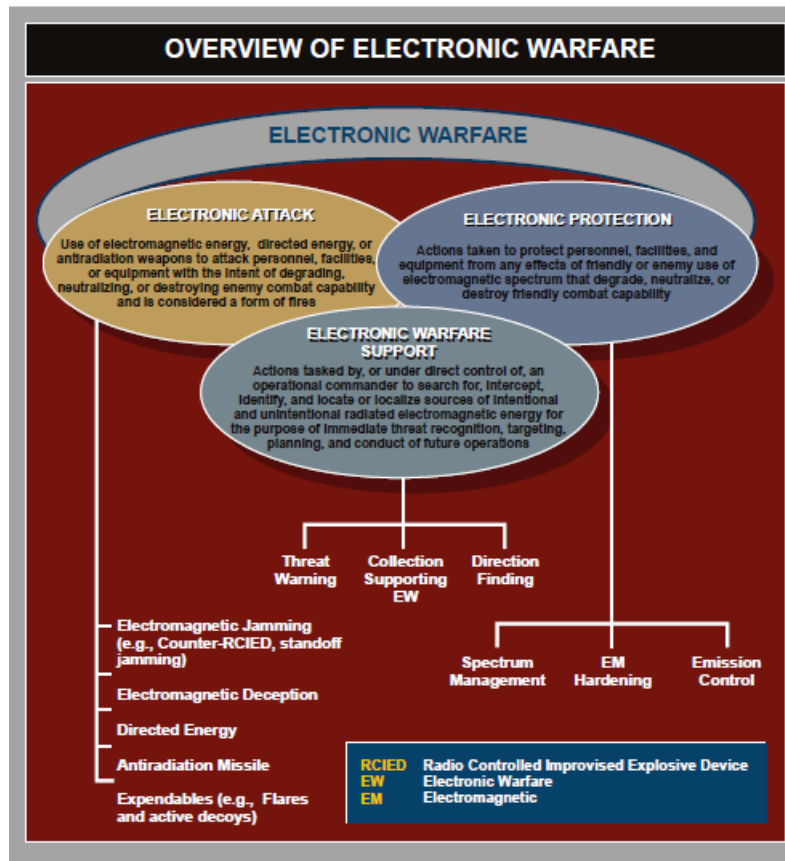
Figure 8.   Overview of EW (From JP 3.13.1, 2007)

Electronic warfare can also be categorized by whether it is considered active or passive. ES tends to be passive, EA tends to be active and EP tends to be both passive and active. Figure 9 shows this relationship. Active activities require emission of detectable signals by the party conducting EW that are transmitted (such as in the example of the jamming of a radar). Passive activities do not emit signals, but rather depend upon detection of signals emitted by a targeted emitter. Active activities can be normally be implemented during peacetime only under strict limitations; on the other hand, passive activities can

be implemented during peacetime with few if any limitations (Frater & Ryan, 2001).
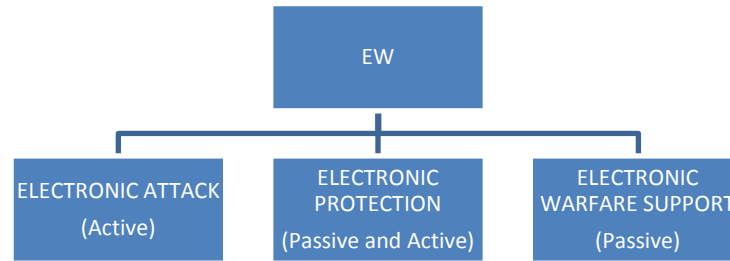
Figure 9.   Categorization of EW based on Active and Passive (Frater & Ryan, 2001)

In this Chapter we have discussed the importance of Electronic Warfare. In the next Chapter we develop the fundamentals of geolocation that will be important to our study.

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III. EMITTER GEOLOCATION

A. INTRODUCTION TO EMITTER GEOLOCATION

Determining the location of a target emitter is one of the fundamental operations of EW, and can serve many useful purposes. The position of an emitter may indicate the position of the enemy forces. In addition, precise location of the target emitter enables the use of Global Positioning Systems (GPS) based weapons.

For civil purposes, knowing the positions of nodes in a wireless network for commercial uses enables a variety of functionalities, such as emergency services, identification and tracking, location dependent computing, health monitoring and geographic routing (Xu, Ma, & Law, 2006).

Passive location finding, in addition to active location finding techniques, can locate stationary and moving targets by measuring the electromagnetic radiation emitted by a target with the added benefit of not having to radiate electromagnetic energy to locate it. Passive and active location finding technologies play important roles in navigation, aviation, aerospace and electronic warfare (Yan-Ping, Feng-Xun, & Yan-Qui, 2010).

The purpose of direction finding (DF) is to estimate or fix the position of selected emitters. This position is not certain because all the measurements include some sort of error, and the entire system includes the noise that is found in all communications systems.

Several techniques can be used to calculate the position of a target emitter. These techniques are based on different types of information acquired from the received signal to calculate a position fix (PF).

The azimuth angle of arrival of a signal, or so called line of bearing (LOB), is the most commonly used technique for calculating a PF. Two or more LOBs are used to determine a position in two dimensions (2D). These LOBs are assumed to be measured at the same time on the same target. These LOBs may

intersect as illustrated in Figure 10. This technique is called triangulation (Poisel, 2005).

Since two LOBs intersect at a point, the information to fix the position of the emitter is not accurate due to measurement and propagation errors. For triangulation, at least three receivers are located on a baseline as illustrated in Figure 10. Each DF receiver has special antennas. These antennas are used to measure the bearings. These bearings are plotted on a map either manually or automatically. Intersection of these bearings (lines) forms a triangle and the possible location of the target is calculated to be at the middle of the triangle. The size of the triangle depends on the accuracy of measurements. The smaller the triangle, the better the accuracy (Frater & Ryan, 2001).
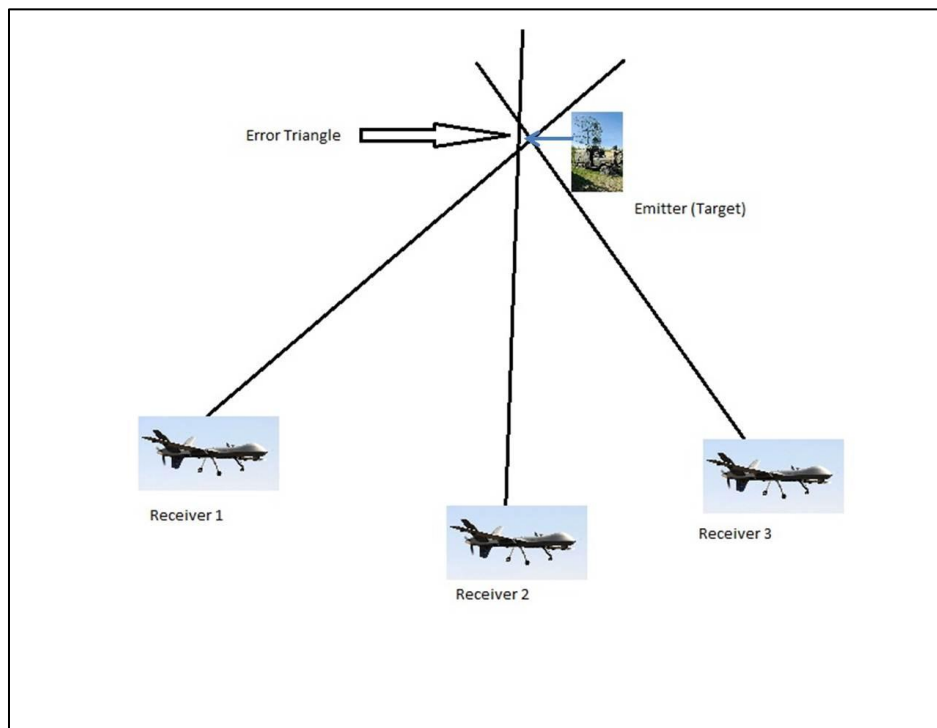

Figure 10.   Intersection of measured LOBs

Another emitter location technique is to measure the time of arrival (TOA) of a signal at several sensors. The TOAs can be used to calculate the position directly, but typically they are sent to a central node where the time difference of

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arrival (TDOA) is calculated from every pair of TOA. Then the range differences between sensors and the target are calculated. These range differences are related to the TDOAs by the speed of propagation in the medium. In the air, this is assumed to be the speed of light.

The TDOA technique generates quadratic lines of position (LOP). All the LOPs are subject to measurement errors and noise. The intersection of these LOPs is used to define the emitter position (Poisel, 2005).

The next step of DF is geolocation. Geolocation is closely related to DF and triangulation, but it is more realistic and distinguished from DF by determining a meaningful location rather than just a set of geographic coordinates.

"Emitter geolocation has two components. One is measurement, or choice of sensors, and the other is estimation/information fusion, or processing of measurements provided by the sensors." (Musicki & Koch, 2008)

Geolocation of a source has a wide variety of applications, such as location of radar sites. Localization of a interference source in satellite communication systems is another example of geolocation (Sathyan, Kriburajan, & Sinha, 2004).

Geolocation is based on techniques that rely on frequency, time, or spatial information, or a combination of these. Common methodologies use angle of arrival (AOA), TOA, TDOA, or differential doppler (DD), also called FDOA (Ho & Chan, 1993) (Loomis, 2007).

B.   BEARING ESTIMATION

Several techniques that can be used to determine the LOPs are discussed in this section. The phase or time difference can be measured between the signals. The amplitude difference between two signals can also be measured. Frequency differences measured can be used to determine the bearing, if one or

more of the receivers are moving relative to the other or to the target (Poisel, 2005).

LOP systems do not operate at the frequencies of the signals. The frequency is typically converted to an Intermediate Frequency (IF) and the phase/time measurements are made on this converted signal (Poisel, 2005).

1. Circular Antenna Array

One of the common types of antenna arrays for bearing determination is a circular array. An example of a four-element array is illustrated in Figure 11. Other forms of this antenna may include more or fewer elements. A sense antenna is included with a circular array. The sense antenna is used to remove ambiguities (Joong, Chul-Gu, & Gyu, 2004).

Let R be the radius of the circular array. R is the length of an "arm" of the array measured from the center to any one of the antenna elements. The *2R/λ* is referred to as the aperture of a circular array (Poisel, 2005).
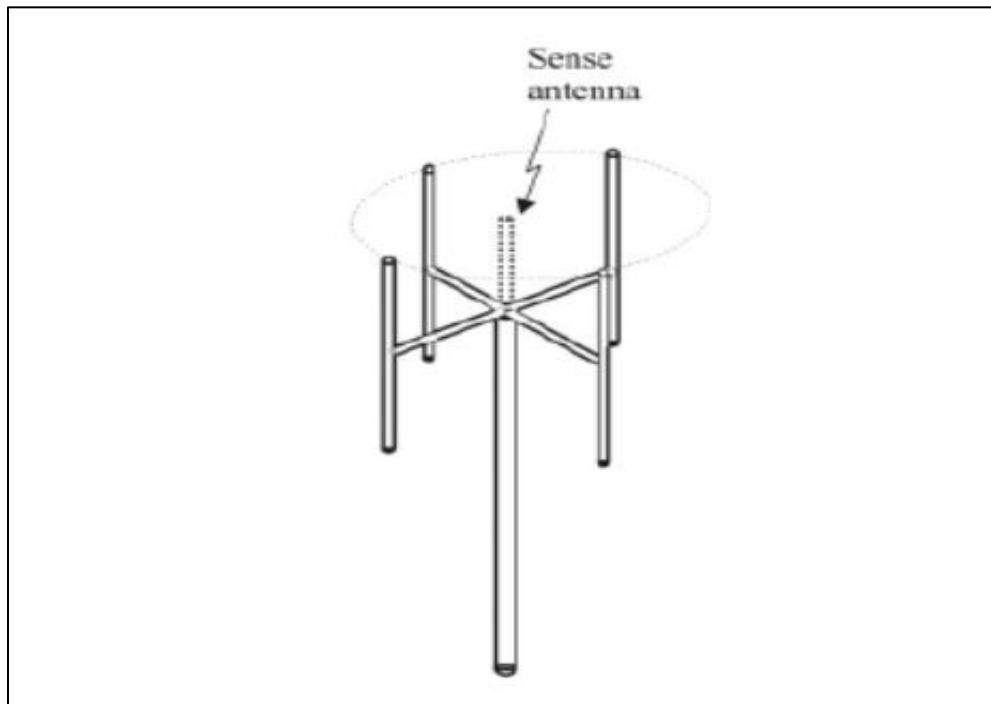
Figure 11.   Four Element Circular Antenna Array (From Poisel, 2005)

An incoming signal *s(t)* and its accompanied E field with a magnitude *E* is illustrated in Figure 12. The vertical component of this *E* field is the only portion that affects a vertically oriented antenna. Thus, E_{vert} is given by $E\cos\alpha$ and the corresponding signal amplitude must be adjusted by this factor (Poisel, 2005).
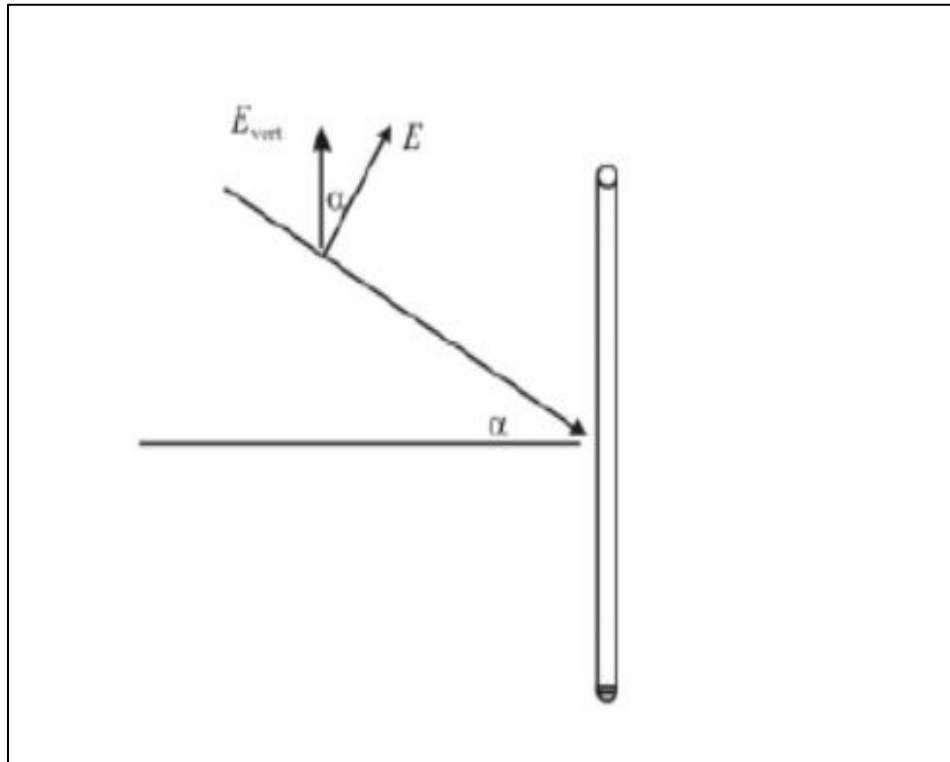
Figure 12.   A signal (s(t)) and the its accompanied E field

2.   Interferometry

One of the techniques for measuring the AOA of a coming signal, and determining its LOP, is interferometry. In interferometry the phase difference or time difference between two antennas is measured directly. The distinction between this technique and other techniques lies in what is measured. Interferometry requires highly accurate measurements. In order to achieve this performance accuracy it is necessary to space at least two of the antennas such that the range of possible phase difference can exceed 2π radsian (Vaccaro, 1993).

Phase comparison DF systems consist of several antenna elements which are arranged in a particular geometric configuration. The number of the elements and the arrangement depends on the DOAs of interest and the method used to process the signals. The minimum number of elements is two. The determination of DOA is performed by direct phase comparison of the received signals from the different antenna elements.

There are two types of interferometers. The first type measures the phase difference between the two antennas and calculates the AOA from that measurement. These interferometers are called phase interferometers. The second type measures the time of arrival differences to the two antennas and calculates the AOA from these measurements. These types of interferometers are called active time interferometers. For this second type of interferometer there must be some sort of time mark to be able to measure the time difference. If there is no time mark then the signal reaching these two antennas must be correlated. Radar pulses provide a convenient time mark in their leading edge.

Interferometer systems operate well over limited frequency ranges. They provide the capability of receiving signals with good accuracy (Lipsky, 1987).

C.   QUADRATIC POSITION FIXING TECHNIQUES

Techniques for determining a position fix based on Time of Arrival (TOA), Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) (also known as Differential Doppler or Differential Frequency) techniques are explained in this section.

When TOA or TDOA is measured at two or more widely separated receivers, quadratic LOPs are determined. The intersection of these LOP curves is taken as the estimated location of the emitter.

The receivers and the emitter might be stationary for TDOA and TOA but for FDOA either the receiver or the emitter must be moving in order to produce a frequency difference induced by movement, necessary for measurement.

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The main advantages of using these techniques are:

- Most of the time only one antenna is used per receiver instead of two or more antennas as in interferometric techniques.

- Higher precision and more accuracy can be obtained with these techniques (Poisel. 2005).

On the other hand, there are two main disadvantages:

- Preprocessed data samples are required for non-pulsed modulation, like Continuous Wave (CW) signals such as using Frequency Modulation or Amplitude Modulation (Poisel, 2005). For pulsed conventional radar signals it is easy to define the beginning of the signal, but for CW signals more complicated methods like correlation should be used.

- Also, it is difficult to measure frequency of pulse-type signals to the level of accuracy needed to do FDOA because the frequency resolution is equal to $1/T$, where T is the pulse duration.

This following section begins with a presentation of the TDOA technique, followed by a discussion of FDOA, and concludes with a discussion of TOA.

1. Time Difference of Arrival (TDOA)

The TDOA involves the measurement of the TOA of the received signal. The TDOA technique needs two or more geographically separated sensors synchronized with each other to be able to find the location of the emitter. If only two receivers are used, this will normally result in an ambiguous solution of two locations, and a third receiver is necessary to resolve this ambiguity. These receivers can be either on the ground or on an airborne platform.

The geometry of TDOA is shown in Figure 13. Only two receivers are shown for simplicity and will be used for calculations. This geometry is for 2D but if the receivers or the emitter is elevated it becomes 3D. For both the scenarios, r is the slant range where r_1 is the range between receiver 1 and emitter and r_2 is the range between receiver 2 and emitter.
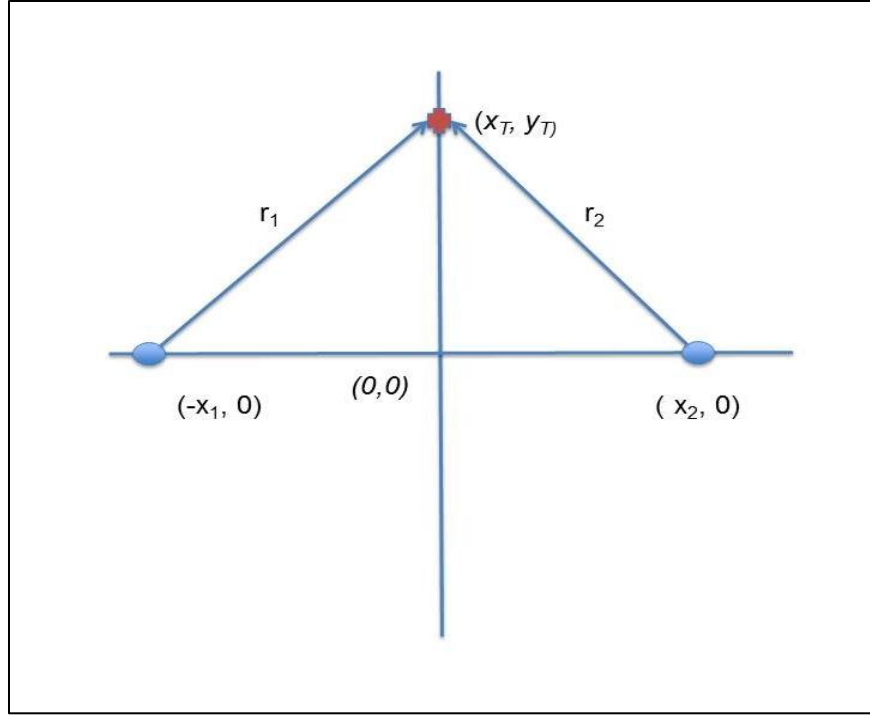
Figure 13.   Geometry of TDOA

The geometry involves an emitter at position $x = (x_T, y_T)$ and two receivers at positions $(-x_1, 0)$ and $(x_2, 0)$, At time $t = 0$, a measurement of the TDOA is made between the arrival of the same pulse from the emitter arriving at the two receivers.

The distances between receivers and the emitter can be calculated as follows:

$$r_i = ct_i \qquad i = 1,2 \qquad\qquad (3.1)$$

where;

c is the speed of light,

t_i is the time between when the signal leaves the emitter and when it arrives the receiver.

TDOA is the time difference between when the signal arrives at one receiving site and at the other, and represented as τ

$$\tau = t_2 - t_1 = \frac{r_2}{c} - \frac{r_1}{c} = \frac{1}{c}(r_2 - r_1) \tag{3.2}$$

$$r_{i=} \sqrt{(x_T - x_i)^2 + y_T^2} \qquad i = 1,2 \tag{3.3}$$

$$\tau c = \delta = \left[\sqrt{(x_T - x_1)^2 + y_T^2} - \sqrt{(x_T - x_2)^2 + y_T^2} \right] \tag{3.4}$$

Squaring both sides of the previous equation yields: (the receivers are at the same distance from the origin)

$$\delta^2 = 2x_T^2 + 2y_T^2 + 2s^2 - 2\sqrt{\left[(x_T + s)^2 + y_T^2\right]\left[(x_T - s)^2 + y_T^2\right]} \tag{3.5}$$

where

$$x_1 = -s \ \& \ x_2 = s$$

With some more algebra, this expression becomes

$$\delta^4 - 4s^2\delta^2 = (4\delta^2 - 16s^2)x_T^2 + 4\delta^2 y_T^2 \tag{3.6}$$

After further manipulation, this becomes:

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$$1 = \frac{x_T^2}{\delta^2/4} - \frac{y_T^2}{(4s^2 - \delta^2)/4} \qquad (3.7)$$

This is familiar equation for a hyperbola, which has x-axis intercept of (0, 0) and is asymptotic to the lines (Loomis, 2007)

$$y = \pm \left(\frac{\sqrt{4s^2 - \delta^2}}{\delta} \right) x \qquad (3.8)$$

The curve defined by this expression is a hyperbola. Several of the hyperbolic curves are shown in Figure 14.
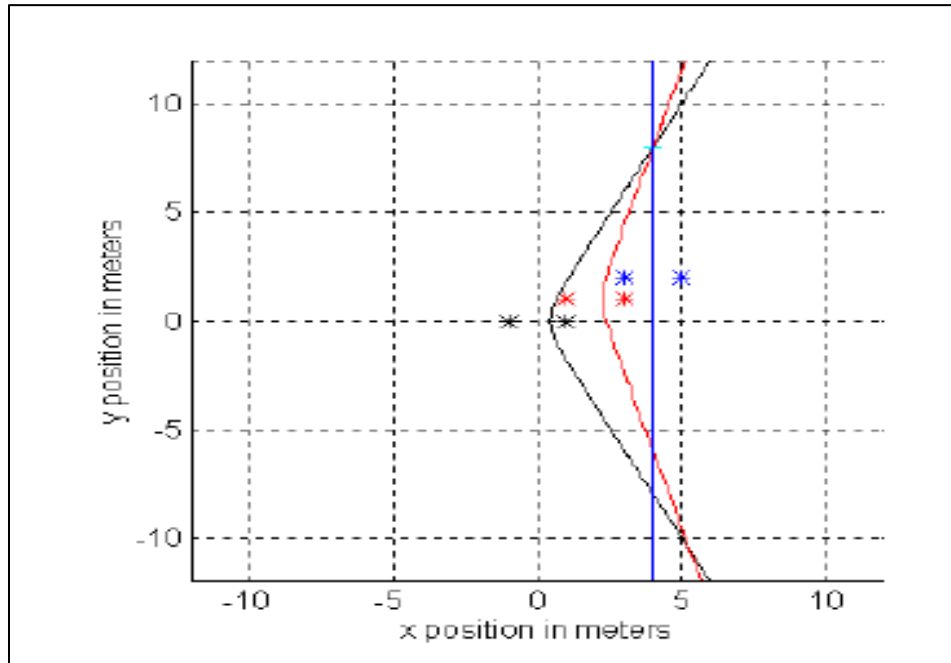
Figure 14.   Hyperbolic TDOA Curves (From Loomis, 2007)

It is clear that two receivers cannot find the location of the emitter since these hyperbolas covers a wide variety of locations. At least three receivers are necessary to find the geolocation of the emitter in two dimensions or on the earth's surface.

The following derivation of the solution for the location of the emitter as a mathematical model of TDOA is quoted from Poisel's book, *Introduction to Communication Electronic Warfare Systems*, Chapter 12.

Suppose there are S receivers available to receive and compute the location of the emitter, then the Equation (3.1) becomes for all pairs of sensors (Poisel, 2002).

$$d_i - d_j = c\left(t_i - t_j\right) = ct_{ij} \qquad\qquad i, j = 0, 1, \ldots S - 1 \qquad (3.9)$$

According to Poisel's approach, let's assume that all of the arrival times are compared with the arrival time at a sensor located at coordinates $(0,0)$ as shown in Figure 15.
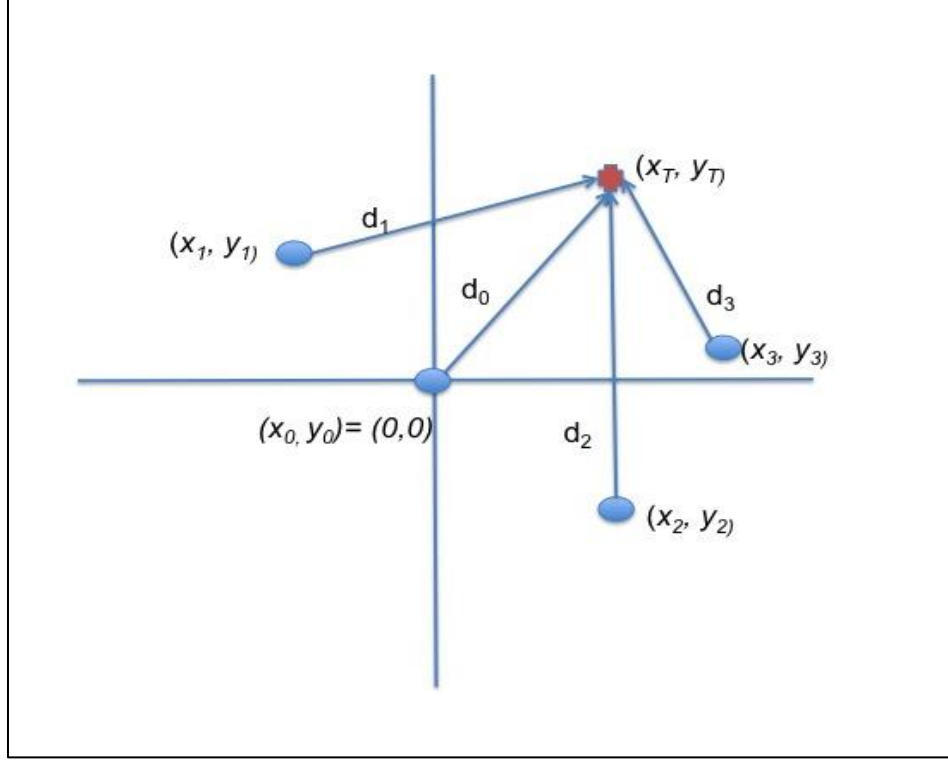
Figure 15.   Sensor Grid and target in Two Dimensions

The time differences of arbitrary (i, j) are not used, just $(i,0)$ is used for all i. The Equation (3.9) in this case reduces to

$$d_i = \|\mathbf{r}_i - \mathbf{r_T}\| = c\left(t_{i,0} + t_0\right)$$

(3.10)

When putting the locations of the emitter and the receivers in to Equation (3.10), it becomes

$$\begin{bmatrix} x_i & y_i & z_i \end{bmatrix} \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} + ct_{i,0}\sqrt{x_T^2 + y_T^2 + z_T^2}$$

(3.11)

$$= -\frac{1}{2}c^2 t_{i,o}^2 + \frac{1}{2}(x_i^2 + y_i^2 + z_i^2)$$

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Putting Equation (3.11) into matrix form

$$\mathbf{p_i x_T} + ct_{i,0} \|\mathbf{x_T}\| = -\frac{1}{2} ct_{i,0}{}^2 + \frac{1}{2} \|\mathbf{p}_i\|^2 \qquad (3.12)$$

where

$\mathbf{p}_i$ is the position vector of receiver i

Expanding this result for all $i = 1...S$ receivers yields

$$\mathbf{P x_T} + \mathbf{c} \|\mathbf{x_T}\| = \mathbf{d} \qquad (3.13)$$

where

$$\mathbf{P} = \begin{pmatrix} x_1 & y_1 & z_1 \\ \vdots & \vdots & \vdots \\ x_{s-1} & y_{s-1} & z_{s-1} \end{pmatrix}$$

$$\mathbf{x_T} = \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix}$$

$$\mathbf{t} = \begin{bmatrix} t_{1,0} \\ \vdots \\ t_{s-1,0} \end{bmatrix}$$

$$\mathbf{c} = c\mathbf{t}$$

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$$\mathbf{d} = \frac{1}{2} diag(\mathbf{PP}^T - c^2 \mathbf{tt}^T)$$

Let $a = \|\mathbf{x_T}\|$ and $\mathbf{Q} = (\mathbf{PP}^T)^{-1}$, then the expression becomes

$$\left(\mathbf{c}^T \mathbf{Qc} - 1\right)a^2 - 2\mathbf{d}^T \mathbf{Qc}a + \mathbf{d}^T \mathbf{Qd} = 0 \qquad (3.14)$$

which is a quadratic equation that can easily be solved for *a* which is the range of the target from the origin. Substituting this range back into Equation (3.13) will solve the problem for the target location.

$$\mathbf{Px_T} + a\mathbf{c} = \mathbf{d}$$

$$\qquad\qquad\qquad (3.15)$$

$$\mathbf{x_T} = \mathbf{P}^{-1}(\mathbf{d} - a\mathbf{c})$$

If S>4, this becomes an over-determined system of equations. Instead of the inverse, then, the pseudo inverse can be used and is shown in Equation (3.16)

$$\mathbf{P}^\dagger = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \qquad (3.16)$$

The pseudo inverse solves for the emitter position in the minimum least squares error sense (Poisel, 2002). It is very important for this thesis and for many of TDOA solutions. The pseudo inverse is used in Chapter 4 for simulation which is a developed version of Ezzat's Closed-Form Geolocation Solution.

2.   Frequency Difference of Arrival-Differential Doppler (FDOA-DD)

The signal emitted by a target of interest which is moving produces an effect called Doppler shift. The Doppler shift is related to the direction of the movement relative to the receiver and shows itself as a frequency difference. The movement can be at the target or at the receiver. Each of these movements produces the same frequency difference effect. When the frequency difference between the target and the receivers of two (the more the receivers the better the calculation) is measured, these measurements can be used to calculate the geolocation of the target. This type of geolocation technique is called frequency difference of arrival (FDOA) or differential Doppler (DD).

Since this thesis is interested in stationary receivers and FDOA technique requires a moving receiver or a moving target, FDOA is not examined in detail.

D.   CLOSED-FORM SOLUTION OF HYPERBOLIC GEOLOCATION EQUATIONS

3D geolocation is important for the Turkish Army because the terrain where most of the anti-terrorist operations take place is very mountainous, especially the southeast part of Turkey. It is not possible to locate or position the receivers at the same elevation with the transmitter. It is inevitable to be presented with a 3D problem. Ezzat's approach gives a unique solution to a 3D problem from the TDOA perspective. Because of that reason, Ezzat's approach is used for developing and analyzing the scenarios in Chapter 4.

The following approach is paraphrased from Ezzat's article.

The approach that is explained here does not depend upon range data. Range data is derived by multiplying the time that the signal propagated in air and the speed of the signal, the speed of light. It is necessary to know the time of transmission and the time of arrival to get the amount of time travelled from the transmitter to the receiver. For the most part, it is impossible to know the time of transmission, which can be defined as t_0. Ezzat indicate in his paper that this is the only technique that can be used without the range data. He mentions that

without t_0, it is possible to find the location of the emitter. He also indicates that the previous solutions are good for a noise free environment.

The closed-form solution presented does not require the calculation of range data and does not depend on the availability of any information other than the times of arrival.

The basic form of time of arrival equation is as follows

$$t_i = t_0 + \frac{D_i}{c}$$ (3.17)

where

t_i is the time of arrival at receiver i,

D_i is the distance between the emitter and the receiver,

c is the speed of light.

t_0 is the time of the transmission

Two or more receivers are needed to be able to calculate the TDOA as mentioned before. When this condition is satisfied, it is possible to eliminate t_0 from any pair of two equations, which results in

$$t_2 - t_1 = \frac{D_2 - D_1}{c}$$ (3.18)

This is the TDOA equation. This equation yields a 3D hyperboloid as mentioned before. When the emitter coordinates, x_0, y_0, and z_0, are plugged into this equation,(3.18) can be written as

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$$\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2} - \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

$$= c(t_2 - t_1)$$

(3.19)

where

x_1, y_1, z_1 and x_2, y_2, z_2 are the coordinates of the receiving antennas 1 and 2.

Having three more receiving antennas yields three additional times of arrival, which produce an additional three equations like Equation (3.19), it is possible to solve for the emitter coordinates x_0, y_0, z_0

There is an effect of path delay on (3.18) and hence(3.19). Ezzat says that a propagation mode between any two points in which the path of the signal is not a straight line will be mathematically equivalent to propagation along a straight line but with a velocity that is less than c, as the time of arrival is important. The TDOA equation for the case in which the path of the signal is nonlinear, it will be written as follows after adding the path delay:

$$t_2 - t_1 = \frac{D_2}{\alpha_2 c} - \frac{D_1}{\alpha_1 c} = \frac{1}{c}(\frac{D_2}{\alpha_2} - \frac{D_1}{\alpha_1})$$

(3.20)

where

α_1 and α_2 are path delay coefficients which are less than or equal to 1.

α is one when there is no path effect on the propagating signal like in the case where the signal propagates through air. For this thesis, all the path delay coefficients are assumed to be one.

The solution that is used for this thesis is derived from Ezzat's article. First, I will explain his approach. Then, because he did not discuss the effects of noise, I will add noise to his approach.

These are the steps to transform the hyperbolic equations into a set of vector equations. In Equation (3.18), we first note that a distance d_i can be written as the norm of a vector.

$$d_i = |\mathbf{p_i} - \mathbf{p_0}| \tag{3.21}$$

where

$\qquad \mathbf{p_i} = (x_i, y_i, z_i)$ $\qquad$ is the position vector of the receiving antenna

$\quad$ and

$\qquad \mathbf{p_0} = (x_0, y_0, z_0)$ $\qquad$ is the position vector of the emitter.

Equation (3.20) represents the difference between the two. We modify Equation (3.20) and write it as a difference of squares

$$(t_2 - t_0)^2 - (t_1 - t_0)^2 = \frac{1}{c^2}(\frac{d_2^2}{\alpha_2^2} - \frac{d_1^2}{\alpha_1^2}) \tag{3.22}$$

From Equation (3.21) and Equation (3.22) we have

$$\frac{\left|\mathbf{p_2} - \mathbf{p_0}\right|^2}{\alpha_2^{\,2}} - \frac{\left|\mathbf{p_1} - \mathbf{p_0}\right|^2}{\alpha_1^{\,2}} = c^2\left(t_2^{\,2} + t_0^{\,2} - 2t_2 t_0 - t_1^{\,2} - t_0^{\,2} + 2t_1 t_0\right)$$

$$= c^2\left(t_2^{\,2} - t_1^{\,2}\right) - 2t_0 c^2(t_2 - t_1)$$

(3.23)

$\left|\mathbf{p_i} - \mathbf{p_0}\right|^2$ can be written as

$$\left|\mathbf{p_i} - \mathbf{p_0}\right|^2 = \left|\mathbf{p_i}\right|^2 + \left|\mathbf{p_0}\right|^2 - 2\mathbf{p_i^T p_0} \tag{3.24}$$

where

$\mathbf{p_i^T}$ represents the transpose of vector $\mathbf{p_i}$.

If we put Equation (3.24) into Equation (3.23) it gives

$$\frac{\left|\mathbf{p_2}\right|^2 + \left|\mathbf{p_0}\right|^2 - 2\mathbf{p_2^T p_0}}{\alpha_2^{\,2}} - \frac{\left|\mathbf{p_1}\right|^2 + \left|\mathbf{p_0}\right|^2 - 2\mathbf{p_1^T p_0}}{\alpha_1^{\,2}} = c^2\left(t_2^{\,2} - t_1^{\,2}\right) - 2t_0 c^2(t_2 - t_1) \tag{3.25}$$

The two coefficients α_1 and α_2 are both very close to one, and as a result the difference $\left|\mathbf{p_2}\right|^2 / \alpha_2^{\,2} - \left|\mathbf{p_1}\right|^2 / \alpha_1^{\,2}$ is negligible by comparison with the other quantities. Then Equation (3.25) becomes

$$\frac{\left|\mathbf{p_2}\right|^2}{\alpha_2^{\,2}} - \frac{\left|\mathbf{p_1}\right|^2}{\alpha_1^{\,2}} - 2\mathbf{p_0}\left(\frac{\mathbf{p_2^T}}{\alpha_2^{\,2}} - \frac{\mathbf{p_1^T}}{\alpha_1^{\,2}}\right) = c^2\left(t_2^{\,2} - t_1^{\,2}\right) - 2t_0 c^2(t_2 - t_1) \tag{3.26}$$

In Equation (3.26) there are two unknowns, t_0 and $\mathbf{p_0}$. To solve the coordinates of the receiver $(\mathbf{p_0})$ we need three linearly independent equations like Equation (3.51). Equation (3.26) shows the relationship between receivers 1 and 2. The other two linearly independent equations can be between 1 and 3 and 1 and 4. This pairing of receivers makes the equations linearly independent. As can be seen for three equations at least four receivers are required. For the closed-form approach, it is necessary to get rid of t_0. To do this these linearly independent equations are going to be coupled to subtract from each other. For that purpose this solution needs five receivers to work.

Equation (3.26) is divided by $(t_2 - t_1)$, then it becomes

$$\frac{1}{(t_2 - t_1)}\left(\frac{|\mathbf{p_2}|^2}{\alpha_2^{\ 2}} - \frac{|\mathbf{p_1}|^2}{\alpha_1^{\ 2}}\right) - \frac{2\mathbf{p_0}}{(t_2 - t_1)}\left(\frac{\mathbf{p_2}^T}{\alpha_2^{\ 2}} - \frac{\mathbf{p_1}^T}{\alpha_1^{\ 2}}\right) = c^2(t_2 + t_1) - 2t_0 c^2 \qquad (3.27)$$

By similar steps Equation (3.27) can be expressed for receiver pairs 3-1 and 4-1

$$\frac{1}{(t_3 - t_1)}\left(\frac{|\mathbf{p_3}|^2}{\alpha_3^{\ 2}} - \frac{|\mathbf{p_1}|^2}{\alpha_1^{\ 2}}\right) - \frac{2\mathbf{p_0}}{(t_3 - t_1)}\left(\frac{\mathbf{p_3}^T}{\alpha_3^{\ 2}} - \frac{\mathbf{p_1}^T}{\alpha_1^{\ 2}}\right) = c^2(t_3 + t_1) - 2t_0 c^2 \qquad (3.28)$$

Equation (3.28) is for 3-1 receiver pair.

$$\frac{1}{(t_4 - t_1)}\left(\frac{|\mathbf{p_4}|^2}{\alpha_4^{\ 2}} - \frac{|\mathbf{p_1}|^2}{\alpha_1^{\ 2}}\right) - \frac{2\mathbf{p_0}}{(t_4 - t_1)}\left(\frac{\mathbf{p_4}^T}{\alpha_4^{\ 2}} - \frac{\mathbf{p_1}^T}{\alpha_1^{\ 2}}\right) = c^2(t_4 + t_1) - 2t_0 c^2 \qquad (3.29)$$

Equation (3.29) is for 4-1 receiver pair.

The next step is to eliminate t_0 from Equations (3.27), (3.28) and (3.29). (3.27) and (3.28) are merged in the components of $\mathbf{p_0}$ to get rid of t_0. This results in

$$\frac{1}{(t_2 - t_1)}\left(\frac{|\mathbf{p_2}|^2}{\alpha_2{}^2} - \frac{|\mathbf{p_1}|^2}{\alpha_1{}^2}\right) - \frac{1}{(t_3 - t_1)}\left(\frac{|\mathbf{p_3}|^2}{\alpha_3{}^2} - \frac{|\mathbf{p_1}|^2}{\alpha_1{}^2}\right) + c^2(t_3 - t_2) =$$

$$\mathbf{p_0}\left[\frac{2}{(t_2 - t_1)}\left(\frac{\mathbf{p_2}^T}{\alpha_2{}^2} - \frac{\mathbf{p_1}^T}{\alpha_1{}^2}\right) - \frac{2}{(t_3 - t_1)}\left(\frac{\mathbf{p_3}^T}{\alpha_3{}^2} - \frac{\mathbf{p_1}^T}{\alpha_1{}^2}\right)\right]$$

(3.30)

Similarly Equations (3.27) and (3.29) yield

$$\frac{1}{(t_2 - t_1)}\left(\frac{|\mathbf{p_2}|^2}{\alpha_2{}^2} - \frac{|\mathbf{p_1}|^2}{\alpha_1{}^2}\right) - \frac{1}{(t_4 - t_1)}\left(\frac{|\mathbf{p_4}|^2}{\alpha_4{}^2} - \frac{|\mathbf{p_1}|^2}{\alpha_1{}^2}\right) + c^2(t_4 - t_2) =$$

$$\mathbf{p_0}\left[\frac{2}{(t_2 - t_1)}\left(\frac{\mathbf{p_2}^T}{\alpha_2{}^2} - \frac{\mathbf{p_1}^T}{\alpha_1{}^2}\right) - \frac{2}{(t_4 - t_1)}\left(\frac{\mathbf{p_4}^T}{\alpha_4{}^2} - \frac{\mathbf{p_1}^T}{\alpha_1{}^2}\right)\right]$$

(3.31)

The last two equations are linearly independent in the components of $\mathbf{p_0}$. As mentioned before at least three independent equations are required. For this purpose there must be another receiver: receiver 5. If we follow the same steps that we followed to reach Equations (3.30) and (3.31), we can have the following equation for receiver-emitter pair 5-1.

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$$\frac{1}{(t_2 - t_1)}\left(\frac{|\mathbf{p_2}|^2}{\alpha_2^{\,2}} - \frac{|\mathbf{p_1}|^2}{\alpha_1^{\,2}}\right) - \frac{1}{(t_5 - t_1)}\left(\frac{|\mathbf{p_5}|^2}{\alpha_5^{\,2}} - \frac{|\mathbf{p_1}|^2}{\alpha_1^{\,2}}\right) + c^2\left(t_5 - t_2\right) =$$

$$\mathbf{p_0}\left[\frac{2}{(t_2 - t_1)}\left(\frac{\mathbf{p_2}^T}{\alpha_2^{\,2}} - \frac{\mathbf{p_1}^T}{\alpha_1^{\,2}}\right) - \frac{2}{(t_5 - t_1)}\left(\frac{\mathbf{p_4}^T}{\alpha_5^{\,2}} - \frac{\mathbf{p_1}^T}{\alpha_1^{\,2}}\right)\right]$$

(3.32)

Equations (3.30), (3.31) and (3.32) can be written in alternative algebraic form

$$a_{11}x_0 + a_{12}y_0 + a_{13}z_0 = b_1$$

$$a_{21}x_0 + a_{22}y_0 + a_{23}z_0 = b_2$$

(3.33)

$$a_{31}z_0 + a_{32}y_0 + a_{33}z_0 = b_3$$

where

$$a_{11} = \frac{1}{(t_2 - t_1)}\left(\frac{x_2}{\alpha_2^{\,2}} - \frac{x_1}{\alpha_1^{\,2}}\right) - \frac{1}{(t_3 - t_1)}\left(\frac{x_3}{\alpha_3^{\,2}} - \frac{x_1}{\alpha_1^{\,2}}\right)$$

$$a_{12} = \frac{1}{(t_2 - t_1)}\left(\frac{y_2}{\alpha_2^{\,2}} - \frac{y_1}{\alpha_1^{\,2}}\right) - \frac{1}{(t_3 - t_1)}\left(\frac{y_3}{\alpha_3^{\,2}} - \frac{y_1}{\alpha_1^{\,2}}\right)$$

(3.34)

$$a_{13} = \frac{1}{(t_2 - t_1)}\left(\frac{z_2}{\alpha_2^{\,2}} - \frac{z_1}{\alpha_1^{\,2}}\right) - \frac{1}{(t_3 - t_1)}\left(\frac{z_3}{\alpha_3^{\,2}} - \frac{z_1}{\alpha_1^{\,2}}\right)$$

$$b_1 = \frac{1}{(t_2 - t_1)}\left(\frac{x_2^2 + y_2^2 + z_2^2}{\alpha_2^2} - \frac{x_1^2 + y_1^2 + z_1^2}{\alpha_1^2}\right)$$

$$-\frac{1}{(t_3 - t_1)}\left(\frac{x_3^2 + y_3^2 + z_3^2}{\alpha_3^2} - \frac{x_1^2 + y_1^2 + z_1^2}{\alpha_1^2}\right)$$

(3.35)

The other constants can be derived from the same Equations (3.31) and (3.32).

Equation (3.33) can be expressed as

$$\mathbf{AX} = \mathbf{B}$$

(3.36)

and can be solved as in least square estimation way as shown in Section E.

$$\mathbf{X} = (\mathbf{A}^T * \mathbf{A})^{-1}\mathbf{A}^T * \mathbf{B}$$

(3.37)

E.   LEAST-SQUARE ESTIMATION

For the least-square estimation, we use Barkat's book, *Signal Detection and Estimation*, 2005.

In the least-square estimation, the criterion is to minimize the squared difference between the given data (signal plus noise) and the assumed signal data.

This development applies to a linearized version of non-linear equations relating an *K*-dimensional measurement vector and the *M*-dimensional vector to be estimated. This linearized matrix equation represents the difference between

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the actual measurement vector and the measurement that would be obtained if the vector to be estimated has the estimated value.

Suppose we want to estimate *M* parameters, denoting the *M*-dimensional vector $\boldsymbol{\theta}$, from the *K* measurements, denoting the *K*-dimensional vector $\mathbf{Y}$ with *K* ≥ *M*. The relation between the parameters $\boldsymbol{\theta}$ and the observed data $\mathbf{Y}$ is given by the linear model

$$\mathbf{Y} = \mathbf{H}\boldsymbol{\theta} + \mathbf{N}$$

where

$\mathbf{H}$ is a known *(K × M)* matrix

$\mathbf{N}$ is the unknown *(K × 1)* error vector that occurs in the measurement of $\boldsymbol{\theta}$.

The least-square estimator of $\boldsymbol{\theta}$ chooses the values that make $\mathbf{X} = \mathbf{H}\boldsymbol{\theta}$ closest to the observed data $\mathbf{Y}$. Hence, we minimize

$$J(\theta) = \sum_{k=1}^{K} (Y_k - X_k)^2 = (\mathbf{Y} - \mathbf{H}\boldsymbol{\theta})^T (\mathbf{Y} - \mathbf{H}\boldsymbol{\theta}) = \mathbf{Y}\mathbf{Y}^T - 2\mathbf{Y}^T\mathbf{H}\boldsymbol{\theta} + \boldsymbol{\theta}^T\mathbf{H}^T\mathbf{H}\boldsymbol{\theta} \quad (3.38)$$

Note that $\mathbf{Y}^T - \mathbf{H}\boldsymbol{\theta}$ is a scalar. Taking the first-order partial derivative of the cost function *J(θ)* with respect to $\boldsymbol{\theta}$ and setting it equal to zero, we obtain the set of linear equations

$$\frac{dJ(\boldsymbol{\theta})}{d\boldsymbol{\theta}} = -2\mathbf{H}^T\mathbf{Y} + 2\mathbf{H}^T\mathbf{H}\boldsymbol{\theta} = 0 \quad (3.39)$$

and LSE can be found to be

$$\hat{\boldsymbol{\theta}}_{ls} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{Y} \qquad (3.40)$$

We observe that the error in the estimator $\hat{\boldsymbol{\theta}}_{ls}$ is a linear function of the measured errors $\mathbf{N}$, since

$$\tilde{\boldsymbol{\theta}}_{ls} \triangleq \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{ls} = \boldsymbol{\theta} - \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{Y} = \boldsymbol{\theta} - \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\left[\mathbf{H}\boldsymbol{\theta} + \mathbf{N}\right] = -(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{N} \quad (3.41)$$

1. Recursive Least-Square Estimator

In real time estimation problems, it is necessary to write the estimator $\hat{\theta}$ in a recursive form for better efficiency. Consider a situation where an estimate $\hat{\theta}$ is determined based on some data $\mathbf{Y_K}$. If new data $\mathbf{Y_{K+1}}$ are to be processed after having determined an estimate based on the data $\mathbf{Y_K}$, it is best to use the old solution along with the new data to determine the new least-square estimator (Barkat, 2005). "It is clear that discarding the estimate based on the data $\mathbf{Y_K}$ and restarting the computation for a solution is inefficient. This procedure of determining the least-square estimate from an estimate based on YK and the new data $\mathbf{Y_{K+1}}$ is referred to as sequential least-square estimation, or more commonly recursive least-square (RLS) estimation." (Barkat, 2005)

Consider the problem of estimating θ from the data vectors $\mathbf{z_M}$ given by the linear model

$$\mathbf{z_M} = \mathbf{h_M}\theta + \mathbf{u_M} \qquad (3.42)$$

where

$$\mathbf{z_M} = [\mathbf{Y_1} \quad \mathbf{Y_2} \quad \dots \quad \mathbf{Y_M}]^T \tag{3.43}$$

is an *(MK+1)* collections of vectors $\mathbf{Y_1}, \mathbf{Y_2}, \dots, \mathbf{Y_M}$ since each vector $\mathbf{Y_k}$, $k = 1, 2, \dots, M$ is a *(K+1)* vector

$$\mathbf{u_M} = [N_1 \, N_2 \dots N_M]^T \tag{3.44}$$

is an *(MK+1)* error vector, and

$$\mathbf{h_M} = [h_1 \, h_2 \dots h_M]^T \tag{3.45}$$

is an *(MK*n)* mapping matrix relating $\mathbf{Z_M}$ to the *(n*1)* parameter vector *θ* to be estimated.

It can be shown that the RLS estimator is given by

$$\hat{\boldsymbol{\theta}}_\mathbf{M} = \hat{\boldsymbol{\theta}}_\mathbf{M-1} + \mathbf{V_M}[\mathbf{u_M} - \mathbf{h_M}\hat{\theta}_\mathbf{M-1}] \tag{3.46}$$

where

$$\mathbf{V_M} = \mathbf{C_{UU}}\mathbf{h_M}^T\mathbf{R_{MM}}^{-1} \tag{3.47}$$

$\mathbf{c}$ is the error covariance matrix given by

$$\mathbf{C_{UU}} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1M} \\ R_{12}{}^T & R_{22} & \cdots & R_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1M}{}^T & R_{2M} & \cdots & R_{MM} \end{bmatrix} \qquad (3.48)$$

In three dimensions, when the covariances are fixed this matrix is

$$\mathbf{C_{UU}} = \begin{bmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y & \rho_{xz}\sigma_x\sigma_z \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 & \rho_{yz}\sigma_y\sigma_z \\ \rho_{xz}\sigma_x\sigma_z & \rho_{yz}\sigma_y\sigma_z & \sigma_z^2 \end{bmatrix} \qquad (3.49)$$

where

σ_x is the variance of x

σ_y is the variance of y

σ_z is the variance of z

ρ_{xy} is the correlation coefficient between x and y

ρ_{xz} is the correlation coefficient between x and z

ρ_{yz} is the correlation coefficient between y and z

If it is assumed that $x,$ $y,$ and z are uncorrelated and so that $\rho_{xy}, \rho_{xz}, \rho_{yz} = 0$. In this case Equation (3.49) becomes (Poisel, 2005)

$$\mathbf{C_{UU}} = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \qquad (3.50)$$

F.   SOURCES OF ERRORS AND MEASUREMENT ACCURACY

1.   Sources of Errors

Direction Finding is subject to number of errors. These error sources are listed and described below.

a.   *Equipment Error*

Modern DF equipment gives bearing with accuracy of $\pm 2^o$. Hand-held tactical units have less accuracy of $\pm 10^o$ (Frater & Ryan, 2001).

b.   *Short-baseline Error*

If the angle between bearing lines is less than 45°, the triangle of error for triangulation or confidence ellipse for hyperbolic geolocation systems like TDOA becomes significantly larger (Frater & Ryan, 2001).

c.   *Co-channel Interference*

Most tactical DF systems cannot identify the difference between multiple received signals. When there is significant co-channel interference these DF systems tend to give erroneous bearing information (Frater & Ryan, 2001).

d.   *Adjacent channel Interference*

"Strong signals in a channel adjacent to the one being DF'ed can lead to an erroneous bearing" (Frater & Ryan, 2001).

e.   *Multipath Error*

In multipath error, two or more signals arrive at the receiver. These received signals originate from the same source but travel in a different path due to natural or man-made obstacles and reach the receiver at different times due to the difference in the distance traveled over the separate paths.

f. Night Effect

Night effect is a special case of multipath effect that occurs when sky-wave propagation occurs at night but not during the day. This type of error occurs at long distance, typically over HF communication channels.

g. Coastal Refraction

Surface wave propagation that crosses a coastline at an angle other than a right angle is subject to bending caused by refraction. This may lead to wrong bearing determination. Coastal refraction is usually significant at frequencies below 10MHz (Frater & Ryan, 2001).

h. Thunderstorms

Thunderstorms can lead to a wrong bearing that points towards the thunderstorm rather than the originating transmitter.

i. Rain

Heavy rain may reduce received signal levels in SHF and higher bands. This reduces the range of DF systems (Frater & Ryan, 2001).

2. Cross-Correlation TDOA Estimation Technique

The TDOA position fixing technique includes two phases. The first phase is the estimation of the TDOAs of the signal from a source, between pairs of receivers through the use of time delay estimation techniques. In the second phase, the estimated TDOAs are transformed into range difference measurements between stations, resulting in a set of hyperbolic equations (Aatique, 1997).

The previous sections related to TDOA calculations are for the second phase.

There are two general methods for estimating the TDOAs. The first one is to subtract TOA measurements from two stations to produce a relative TDOA.

The second one is to employ a cross-correlation technique, in which the received signal in one station is correlated with the received signal at another station.

Because it is very difficult to know the timing reference on the source to be located, and because the signals of interest for this thesis are CW, the cross correlation technique is commonly used to estimate the TDOAs. The basic timing requirement for this technique is to synchronize the receivers. This requirement is relatively easy compared to the need to know the originating transmission time of the signal. Therefore, we will focus on the cross correlation TDOA estimation technique.

Signal, $s(t)$, emanating from a remote source through a channel with noise, the general model for the time-delay estimation between received signals at two base stations, $x_1(t)$ and $x_2(t)$, is given by (Knapp & Carter, 1976)

$$x_1(t) = s(t) + n_1(t)$$

$$x_2(t) = As(t - \tau) + n_2(t)$$

(3.51)

where

$n_1(t)$ and $n_2(t)$	are noises
τ	is the TDOA between the receivers
A	is the amplitude ratio for scaling the signal

This model assumes that $s(t)$, $n_1(t)$ and $n_2(t)$ are real and jointly stationary random processes. The signal $s(t)$ is assumed to be un correlated with noise $n_1(t)$ and $n_2(t)$.

The cross correlation of this two received signal is given by (Knapp et al, 1972)

$$R_{x_1 x_2}(\tau) = E\left[x_1(t)x_2(t-\tau)\right] \tag{3.52}$$

where E represents the expectation. Equation 3.51 can also be expressed as (Aatique, 1997)

$$R_{x_1 x_2}(\tau) = R^0{}_{x_1 x_2}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2(t-\tau)dt \tag{3.53}$$

Because the observation time cannot be infinite and can be estimated from a finite observation time, an estimate of the cross-correlation is given by

$$R_{x_1 x_2}(\tau) = \frac{1}{T}\int_0^T x_1(t)x_2(t-\tau)dt \tag{3.54}$$

where T represents the observation interval. The time delay causing the cross correlation peak is an estimate of TDOA, τ.

The cross correlation technique is affected by many errors, which can be considered as a Gaussian distribution. Standard deviation for that Gaussian distribution is explain in Section 3.

3.   Standard Deviation

When errors producing ambiguity are taken into consideration, TDOA isochrones are no longer clear functions; they form regions or areas where the target should be, as illustrated in Figure 16.

All of the TDOA methods are subject to errors in measurement. The noise and the measurement error are the two primary error sources (Poisel, 2005). For

the purpose of this thesis, errors discussed from this point forward will be limited to noise-induced errors, and they are discussed in terms of standard deviation.
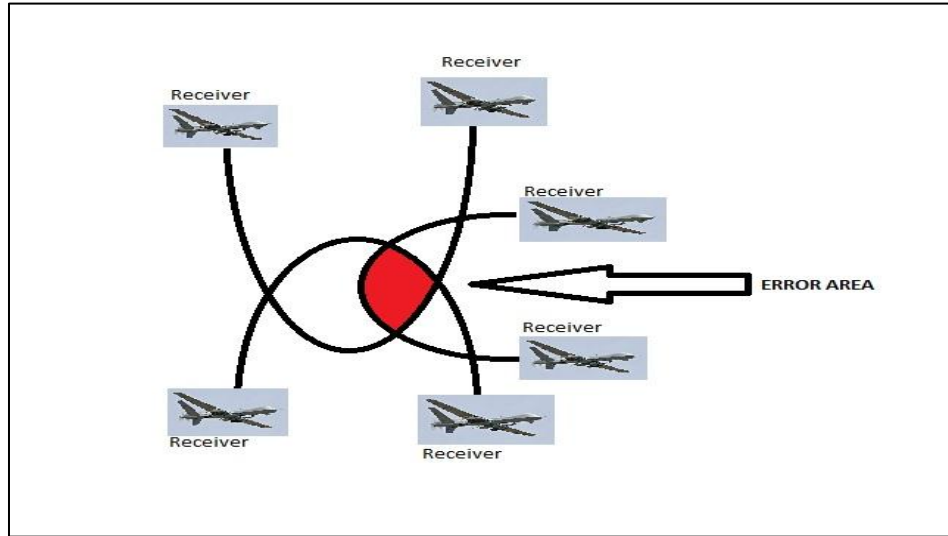

Figure 16.   TDOA Isochrones with Errors

The represented TDOA hyperbolic isochrones can be shown in detail as in Figure 17. The standard deviation increases close to the edges of the hyperbolic isochrones.
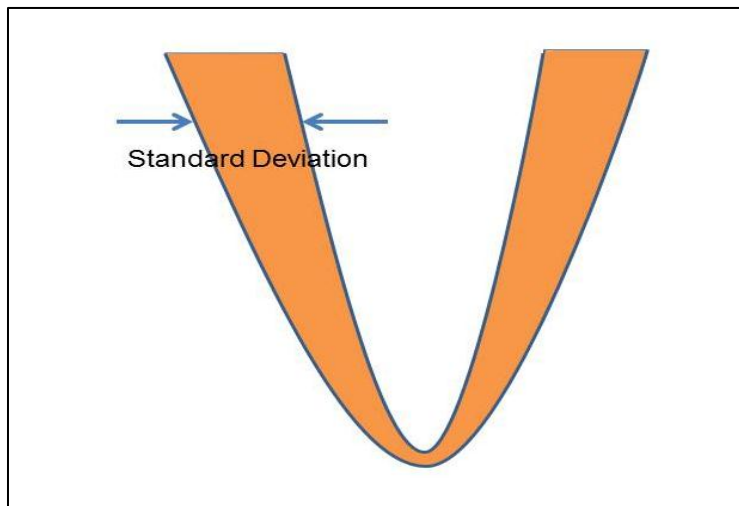

Figure 17.   TDOA Isochrones with Standard Deviation

The standard deviation discussed in this chapter result from cross-correlation measurements.

The Cramer-Rao bound on parameter estimation is a frequently used measure on how well such a parameter can be measured. The Caramer-Rao bound for estimating the time of arrival of a signal at a receiver is given by (Stein, 1981).

$$\sigma_\tau = \frac{1}{\beta}\frac{1}{\sqrt{BT\gamma}} \tag{3.55}$$

where

        B is noise bandwidth of receivers

        T   is integration time, which must be less than or equal to signal duration

        γ is the effective input SNR at two sensor sites

RMS radian frequency is given by β which is the measure of the bandwidth of the signal an given by

$$\beta = 2\pi\left[\frac{\int_{-\infty}^{\infty} f^2\left|S(f)\right|^2 df}{\int_{-\infty}^{\infty}\left|S(f)\right|^2 df}\right]^{\frac{1}{2}} \tag{3.56}$$

where

        $S(f)$ is the spectrum of the signal

Variable γ is a composition SNR at the two sensors and is given by

$$\frac{1}{\gamma} = \frac{1}{2}\left[\frac{1}{\gamma_i} + \frac{1}{\gamma_j} + \frac{1}{\gamma_i\gamma_j}\right] \tag{3.57}$$

where

γ_i and γ_j are the Signal to Noise Ratio (SNR) at two receivers.

For low SNR, standard deviation is given by

$$\sigma_\tau \approx \sqrt{\frac{1}{8\pi^2}} \frac{1}{\delta} \frac{1}{\sqrt{TW}} \frac{1}{f_0} \frac{1}{\sqrt{1 + \frac{W^2}{12f_0^2}}} \tag{3.58}$$

where

T   is the integration time

δ is the SNR

W   is the bandwidth and is given by $W = f_2 - f_1$

f_0 is the center frequency

For high SNR, standard deviation is given by

$$\sigma_\tau \approx \sqrt{\frac{1}{4\pi^2 T}} \frac{1}{\delta} \frac{1}{\sqrt{f_2^3 - f_1^3}} \tag{3.59}$$

The standard deviation function for low SNR is illustrated in Figure 18 and Figure 19 for short integration time and high integration time, and in Figure 20

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and Figure 21 for high SNR for the 25MHz bandwidth (Poisel, 2005). The dotted lines represent the function in Equation (3.55) and the solid lines represent the function in Equation (3.59).
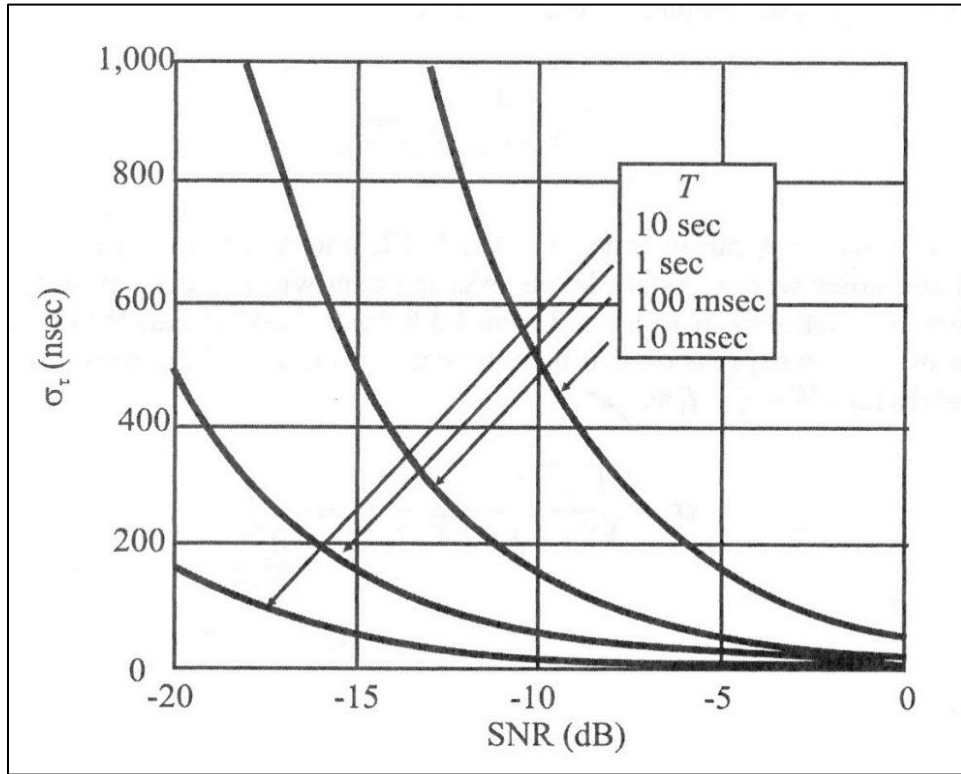
Figure 18.   TDOA Standard Deviation for Low SNR, W=25 MHz and Long Initegration Time (From Poisel, 2005)
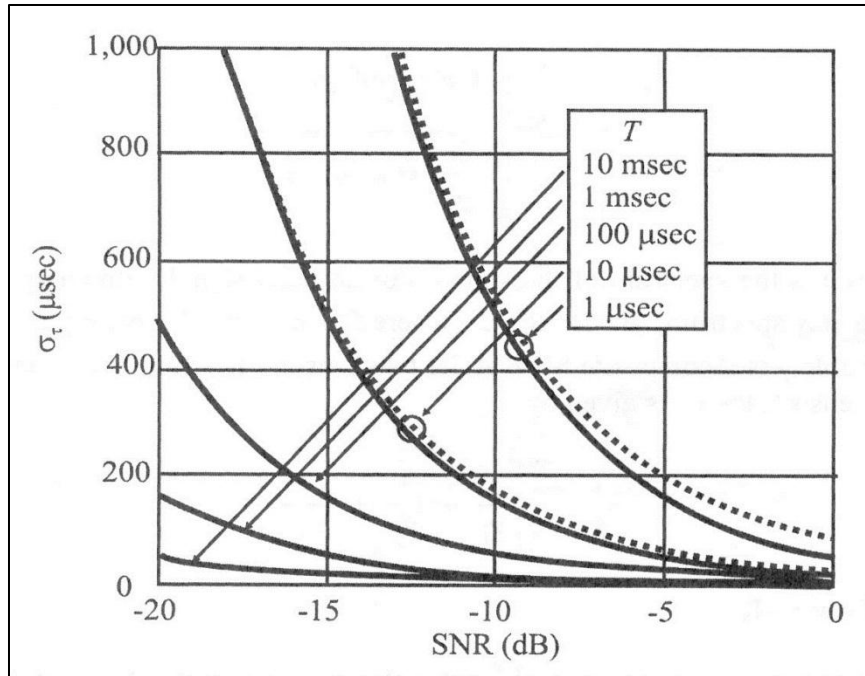
Figure 19.   TDOA Standard Deviation for Low SNR, W=25 MHz and Short Integration
Time (From Poisel, 2005)
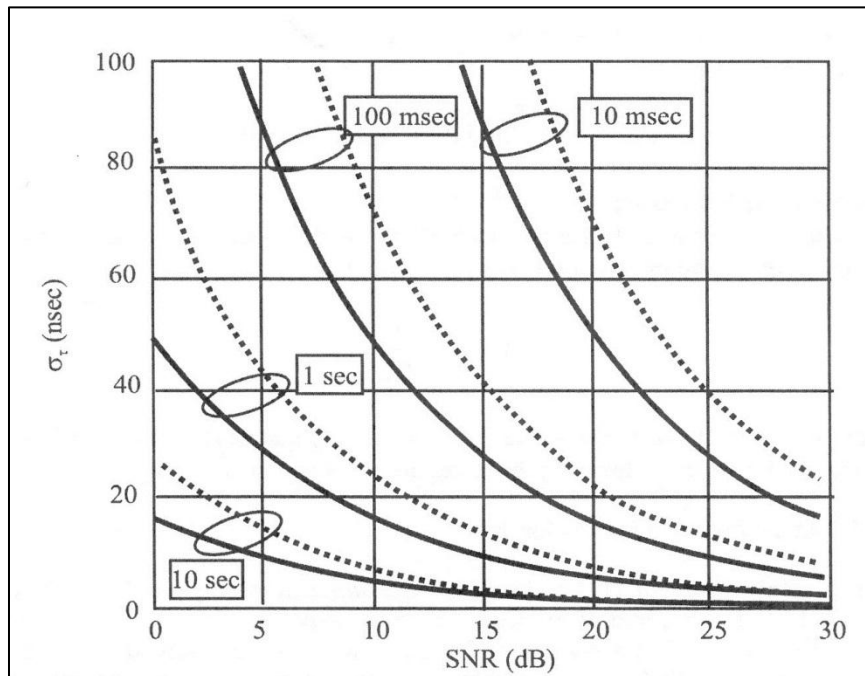
Figure 20.   TDOA Standard Deviation for High SNR, W=25 MHz and Long Integration
Time (From Poisel, 2005)
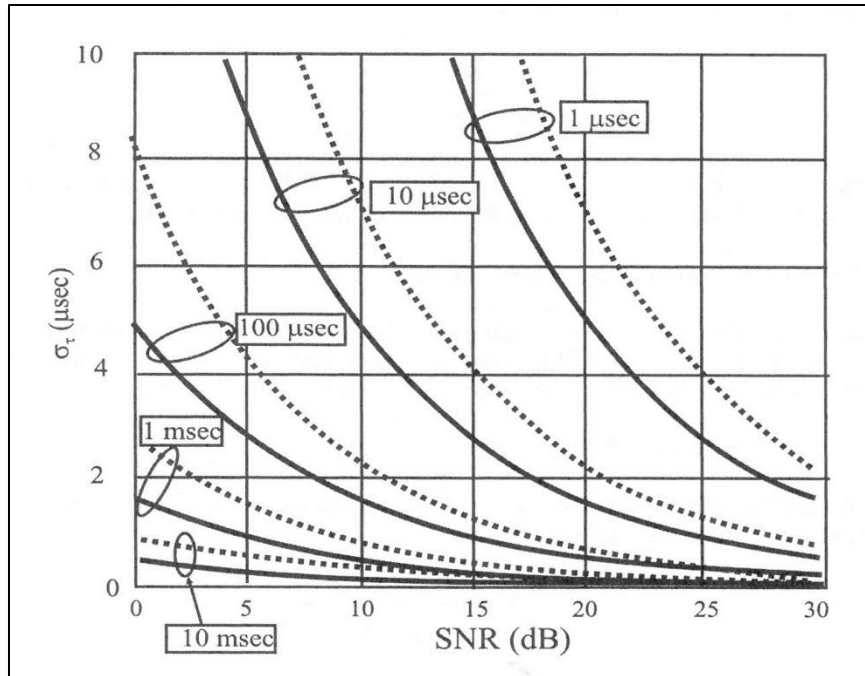
Figure 21.   TDOA Standard Deviation for High SNR, W=25 MHz and Short Integration Time (From Poisel, 2005)

4.   TDOA Dilution of Precision

TDOA measurement suffers from another type of error, caused by the long ranges from the sensor baseline. Consider the Figure 22 where the target is very far from the baseline between the sensors. The hyperbolic LOPs are nearly parallel to each other and make the measurement more difficult, thereby making the system vulnerable to any measurement or noise error. This is called geolocation dilution of precision (GDOP)(Poisel, 2005).
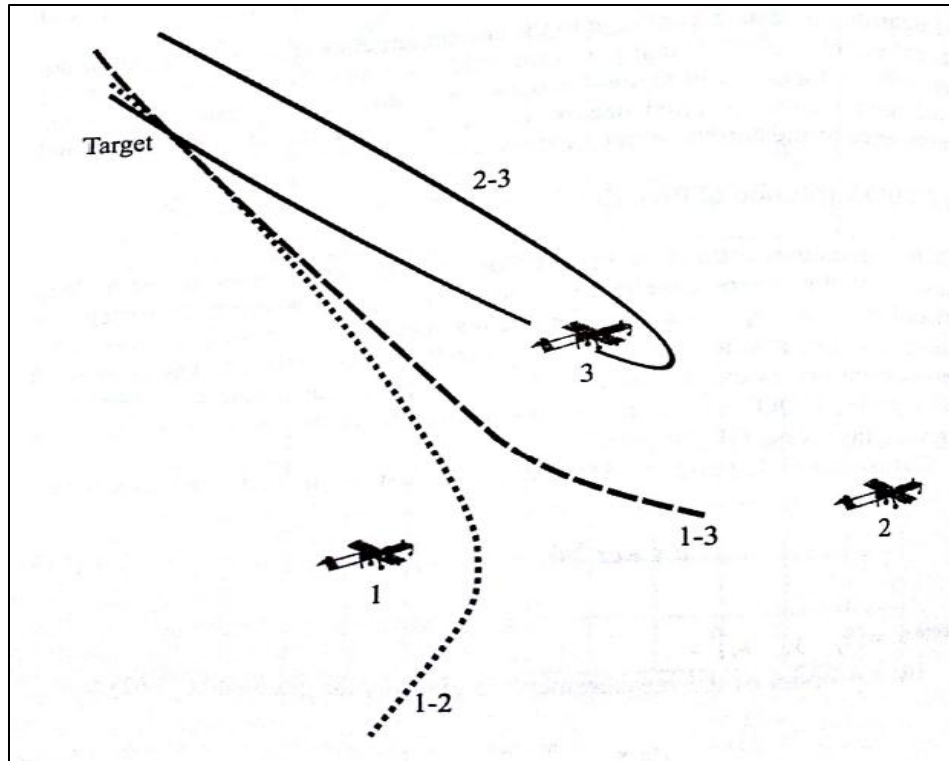
Figure 22.   Geometry of GDOP Effect on TDOA (Poisel, 2005)

The GDOP effect is illustrated in Figure 23 where the effect of the distance to the error can be seen easily. N represents the number of receivers in the system.
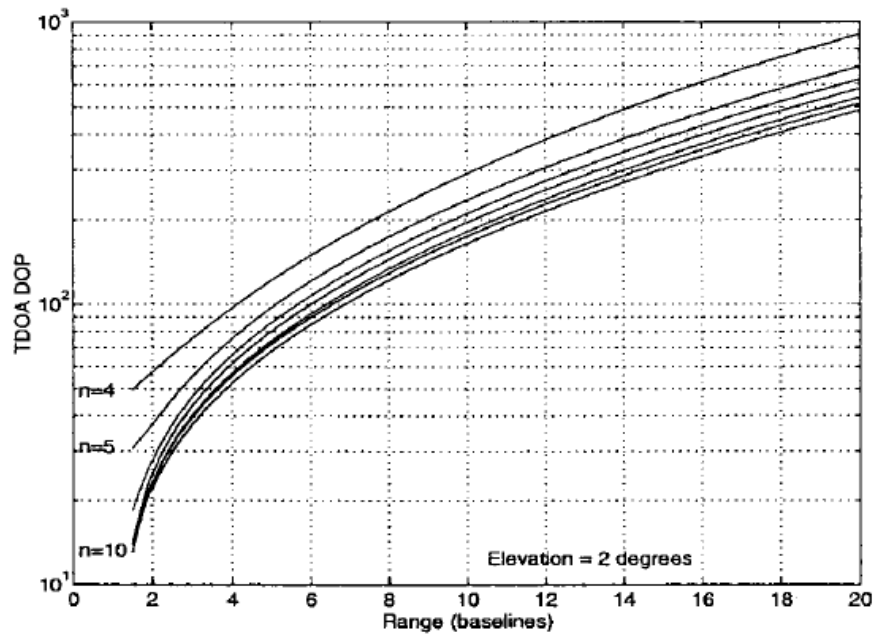
Figure 23.   TDOA GDOP from 4 to 10 elements (From Bard & Ham, 1999)

5.   Effects of Movement on TDOA

When there is a motion between the receivers or the target the error of TDOA measurements increases. This is illustrated by Chan and Ho, who called this effect scale difference of arrival (SDOA). Figure 24 shows the relationship between the velocity and the error caused by this velocity. As can be seen, when the velocity increases the mean square error also increases.
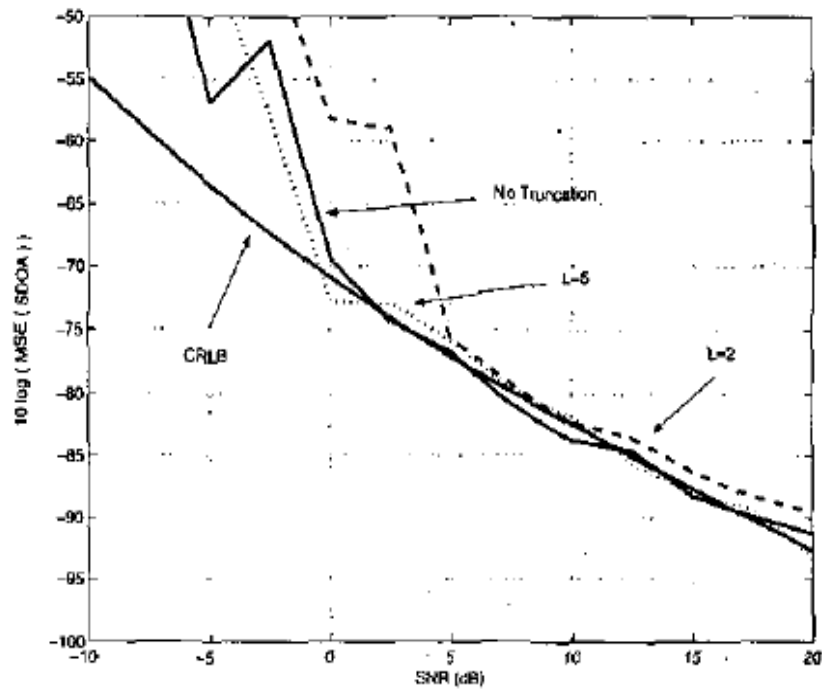
Figure 24.   SDOA Caused by the Relative Motion (From Chan & Ho,2003)

In this Chapter, we discussed the fundamentals of emitter geolocation, bearing estimation, quadratic position finding methods, closed-form geolocation of emitter based on TDOA, least-square estimation method and sources of error for geolocation. In the next chapter, we developed a Matlab[1] simulation based on Ezzat's closed-form geolocation technique and made four type of analysis; the effect of the distance and noise on accuracy, comparison between stationary receivers and moving receivers based on their effects on accuracy, the optimum distribution of stationary receivers for best accuracy.

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[1] Matlab is a registered trademark of The Mathworks, Inc.

IV.   SIMULATION OF THE CLOSED-FORM GEOLOCATION TECHNIQUE

The simulation described in this chapter focuses on the use of time difference of arrival (TDOA) employing Ezzat's proposed closed-form solution for the emitter position in three dimensions (Ezzat, 2007). We will analyze its capabilities in terms of the effect of the number of experiments on accuracy, the effect of a noisy environment on accuracy, the effect of distance between emitter and receivers on accuracy, and the optimum positions of the receivers for best accuracy. Finally, we will make a comparison between moving receivers and stationary receivers.

The assumptions of and the restrictions to the simulation are given in Section A. The development of the simulation and the followed procedures are explained in Section B. Simulation results are given in Section C.

Ezzat's closed-form geolocation technique has been used to develop this simulation. This technique is explained in Section.D in detail. Matlab has been used for simulation, and Matlab code can be found in Appendix A.

In short, simulation is done to accomplish these analyses:

- The effect of number of experiments on the accuracy of the estimated emitter position,
- The effect of the distance between emitter and receivers on the accuracy,
- The effect of the noise on the accuracy,
- Comparison between moving receivers and the stationary receivers,
- The optimum geographical distribution of the receivers for best accuracy.

Different codes are developed for each analysis and can be found in the Appendices.

A.   ASSUMPTIONS AND RESTRICTIONS

The following assumptions and restrictions apply to the development of the technique.

1.   Assumptions

1) The sensor locations are known exactly.

2) The transmitter location is fixed during the period of DF fixing.

3) The sensors are properly located and operated.

4) The emitter is within the range of the receivers.

5) TDOA is measured using some sort of technique and known.

6) The error in the noise is distributed as a Normal distribution with zero mean and known standard deviation.

7) The noise for each pair is independent.

8) There is no multipath effect.

9) The emitter location estimate error is distributed as a normal distribution.

Assumption #1 is reasonable based on the fact that any such position errors can be added to the emitter estimate uncertainty, if they are significant.

Assumption #2 is necessary to the analyses of the systems considered in this thesis, and it is reasonable over the period required to obtain a single fix. Further, any change in emitter position is not significant in comparison to the speed of signal transmission (speed of light) and time to calculate solutions.

Assumption #3 is reasonable in the absence of contradictory information.

Assumption #4 is reasonable based on the fact that if the emitter is out of the range, then the system will not function.

Assumption #5 is reasonable because the measurement of TDOA is out of the interest area of this thesis.

Assumption #6 is reasonable based on the nature of the noise in every receiver system.

Assumption #7 is reasonable based on the fact that every receiver pair has different SNR level, integration time and bandwidth.

Assumption #8 is reasonable based on the fact that multipath effect is out of the interest area of this thesis.

Assumption #9 is usual when considering measurements which are subject to random measurement error. There are biases in the measurements from navigation errors, errors in the calibration tables, interference, etc.; these biases may be removed. In the absence of specific knowledge about these errors the normal assumption is reasonable. Biases will not be treated in this thesis.

2. Restrictions

In addition to the assumptions discussed in this section, this thesis does not consider the following effects;

1) Geographic transformation, map projection effects, and grid reference system conversions.

2) Propagation effects.

3) Susceptibility to deception.

4) Special problems associated with low-probability-of-intercept emitters (low SNR, spread-spectrum, time-frequency diversity, frequency agility, etc.).

5) Numerical computation and normal truncation effects.

6) Equipment errors.

7) Interference.

8) Night effect.

9) Coastal refraction.

10) Thunderstorms and precipitation.

B.   DEVELOPMENT OF THE SIMULATION

Ezzat's technique and Matlab code solve for the position of the emitter directly. The Matlab code includes multipath effect for various cut-off frequencies. The scope of this thesis does not cover the multipath effect, so multipath effect has been removed from the code and all the alphas are considered as one.

Ezzat's solution does not give an error ellipsoid for position estimate error, nor does it yield emitter position-covariance as a function of TDOA measurement error.

The simulation of this thesis is based on Ezzat's simulation. In addition to his simulation, position estimate error is based on multiple simulation runs to get position estimate covariance.

The code of simulation is modified as in Appendix B to decide the number of experiments for minimum estimation error. The total standard deviation of the estimated position for multiple experiments is computed in Equation (3.77). The result is as in Figure 25.
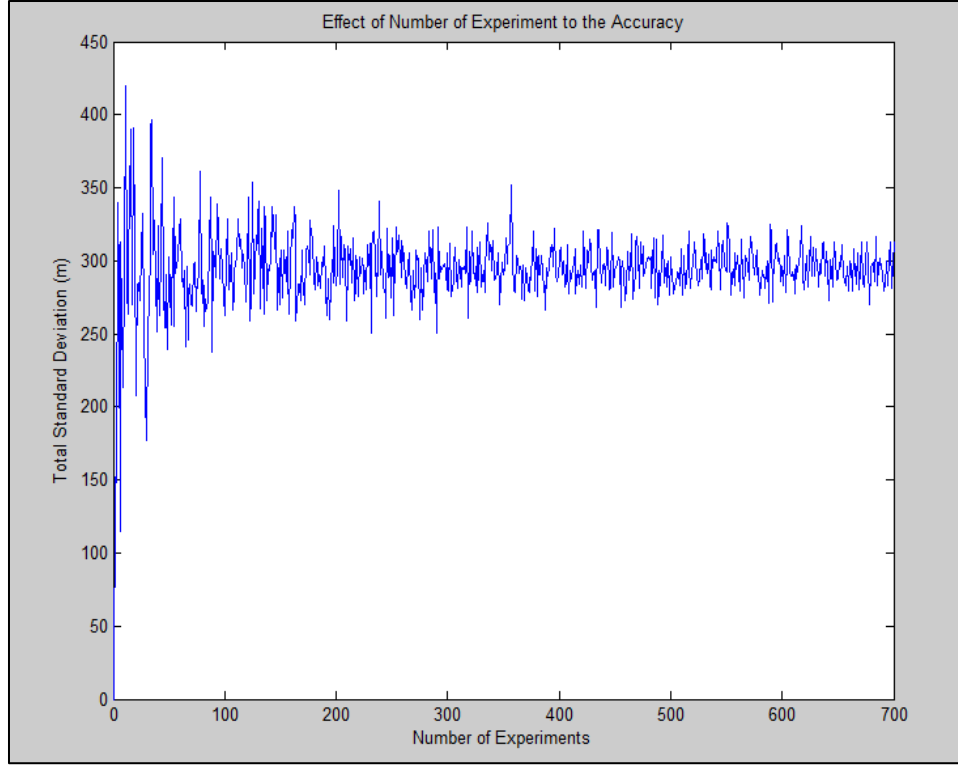
Figure 25.   The Effect of the Number of Experiments on Accuracy

It can be seen from Figure 25 that the total standard deviation reaches a stable level around 500 experiments.

The covariance matrix is based on the error resulting from the noise.

Ezzat's technique does not show the effect of the noise. The effect of the noise to TDOA has been added to this technique and simulation as in

$$TDOA = t + e_N \qquad\qquad (3.60)$$

where

        t   is the actual time difference of arrival

      e_N is the error caused by the noise (Noise Error)

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e_N is a random number from a normal distribution with a 0 mean and σ_τ standard deviation. Standard deviation is explained in detail in Section C Part 2 and the values used for the simulation are taken from that part. Because bandwidth, SNR and integration time changes for every receiver pair, noise error is different for each receiver pair, although the TDOA standard deviation is assumed to be the same for each receiver pair.

The simulation is done for 500 times for efficiency as seen in Figure 25. Each result of the experiments for emitter location error is collected in an error matrix with a size of 500*3. 500 represents the number of experiments and 3 represents the location estimation error for each axis. This error matrix is used to compute the covariance matrix by using Matlab.

Covariance matrix can be found as

$$\text{cov}(X) = E[(X - E[X])(X - E[X])^T] \tag{3.61}$$

If Equation (3.61) is applied to multiple experiments, the covariance matrix consists of variances and correlation coefficients for every axis as seen in Equation (3.62).

$$C_{UU} = \begin{bmatrix} \sigma_x^{\;2} & \rho_{xy}\sigma_x\sigma_y & \rho_{xz}\sigma_x\sigma_z \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^{\;2} & \rho_{yz}\sigma_y\sigma_z \\ \rho_{xz}\sigma_x\sigma_z & \rho_{yz}\sigma_y\sigma_z & \sigma_z^{\;2} \end{bmatrix} \tag{3.62}$$

The covariance matrix shows the relationship between every axis (x, y, and z). Because the covariance matrix shows the relationship between all axis, we convert to a covariance matrix which shows the only the variance for each single axis as in

$$C_{UU} = \begin{bmatrix} \sigma_x{}^2 & 0 & 0 \\ 0 & \sigma_y{}^2 & 0 \\ 0 & 0 & \sigma_z{}^2 \end{bmatrix} \qquad (3.63)$$

The covariance matrix in Equation (3.63) can be interpreted as a confidence ellipsoid. Standard deviation in every axis of Equation (3.63) represents half of the length of each axis of confidence ellipsoid. $2\sigma_x$ represents the length of the x-axis, $2\sigma_y$ represents the length of the y-axis, and $2\sigma_z$ represents the length of the y-axis of the ellipsoid.

Because the covariance matrix in Equation (3.62) shows the noise correlation between every axis, it has to be converted to a diagonalized covariance matrix as in Equation (3.63) in order to get the relationships just for axes with themselves, which can be interpreted as standard deviations along three orthogonal position axes.

Matlab command is used to get the eigenvalues of the covariance matrix. The eigenvalues matrix is the rotation matrix, which can be applied to covariance matrix in Equation (3.62) to get the diagonalized covariance matrix in Equation (3.63). The following computation is done to apply the rotation matrix to the covariance matrix (3.62) in order to get the diagonalized covariance matrix in (3.63)

$$\textbf{DiagonalCovMat} = \textbf{(EigenVectors)'} * \textbf{(CovMat)}^{-1} * \textbf{EigenVectors} \quad (3.64)$$

The rotation matrix is explained in Slabaugh and Shoemake's paper (Slabaugh, 1999 & Shoemake, 1985). The rotation has been done according to the following order; rotate x axis, rotate y axis and rotate z axis.

$$\mathbf{R} = \mathbf{R}_z(\phi)\mathbf{R}_y(\theta)\mathbf{R}_x(\psi) \tag{3.65}$$

where

$\mathbf{R}_z(\phi)$ is the rotation in z axis with the angle of ϕ

$\mathbf{R}_y(\theta)$ is the rotation in y axis with the angle of θ

$\mathbf{R}_x(\psi)$ is the rotation in y axis with the angle of ψ

Rotation for z axis is defined as follows

$$\mathbf{R}_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.66}$$

Rotation for y axis is defined as follows

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{3.67}$$

Rotation for x axis is defined as follows

$$\mathbf{R}_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \tag{3.68}$$

As a result rotation matrix is

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \tag{3.69}$$

and in terms of rotation angle

$$\mathbf{R} = \begin{bmatrix} \cos\theta\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \cos\theta\sin\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi \\ -\sin\theta & \sin\psi\cos\theta & \cos\psi\cos\theta \end{bmatrix} \tag{3.70}$$

where the rotation angles are

$$\theta = -\sin^{-1}(R_{31}) \tag{3.71}$$

$$\psi = a\tan 2\left(\frac{R_{32}}{\cos\theta}, \frac{R_{33}}{\cos\theta}\right) \tag{3.72}$$

$$\phi = a\tan 2\left(\frac{R_{21}}{\cos\theta}, \frac{R_{11}}{\cos\theta}\right) \tag{3.73}$$

The rotation matrix has to be orthogonal; in other words, $\mathbf{R}*\mathbf{R}^\mathsf{T} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix or $\mathbf{R}^{-1} = \mathbf{R}^\mathsf{T}$, where $\mathbf{R}^{-1}$ is inverse of rotation matrix and $\mathbf{R}^\mathsf{T}$ is the transpose of the rotation matrix. The easiest way to find the rotation matrix is to find the eigenvectors of the covariance matrix. A matrix formed from the combination of the eigenvectors of the covariance matrix is the rotation matrix.

As explained earlier, the result in Equation (3.63) gives the variances for each axis diagonally. The square root of these variances gives the standard

deviation that is essential to check the accuracy of the system. An ellipsoid represents where the estimated emitter location is; the estimated location of the emitter might be any point inside that ellipsoid. These variances form this ellipsoid. Using the square root of each variance, standard deviations represent the length of the half of each axis. This ellipsoid represents the volume of space which contains the emitter location with probability 0.25.

A 25 percent probability represents one standard deviation. We need to multiply these standard deviations to change the scale of the ellipsoid in order to add different probabilities. Ezzat's algorithm and simulation do not include any probability of the estimated emitter location.

The probability is added to the simulation as the length of the each semi-axis of the ellipsoid as explained in Torrieri's paper as (Torrieri, 1984)

$$l_{x-axis} = \sqrt{\kappa \sigma_x^2}$$

$$l_{y-axis} = \sqrt{\kappa \sigma_y^2} \qquad (3.74)$$

$$l_{z-axis} = \sqrt{\kappa \sigma_z^2}$$

The probability of the actual location of the emitter lies inside that ellipsoid is given by (Torrieri, 1984)

$$P_e(\kappa) = erf(\sqrt{\kappa}/2) - \left(\sqrt{2\kappa}/\pi\right)\exp(-\kappa/2) \qquad (3.75)$$

where

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-t^2)dt \qquad (3.76)$$

If the reader is more interested in the statistical theory, he/she can read Torrieri's paper (Torrieri, 1984).

κ values have been calculated for various probabilities and shown in Table 1.

Probability	κ
99 %	13.89702713
95 %	8.934259824
90 %	6.922544695
85 %	5.766298681
80 %	4.949459443
75 %	4.314831079
50 %	2.298290951
45 %	2.008418735
25 %	1.011546612

Table 1.   Probability of Actual Location of Emitter to Lie in Ellipsoid and κ Values

The κ values become larger when the probability increases, resulting in a larger ellipsoid as seen in Figure 26.

Figure 26. κ Values

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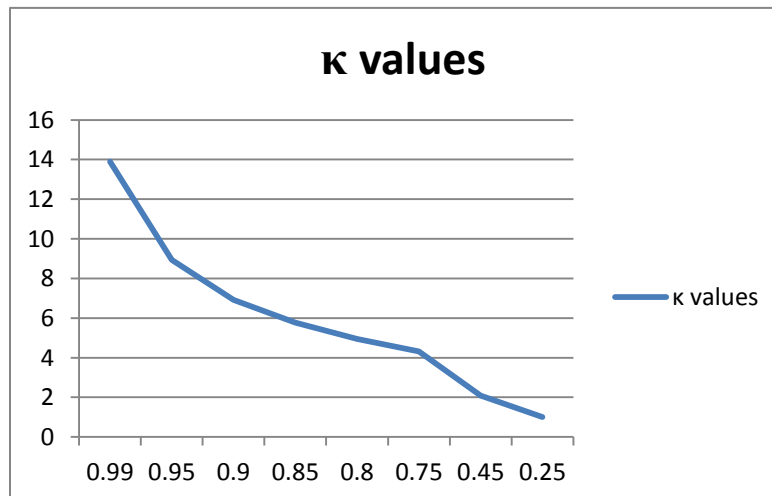
C.   SIMULATION RESULTS

Since the standard deviation gives a good understanding of the accuracy, the simulation is run to analyze the accuracy with the standard deviation for every axis and the total standard deviation.

Total standard deviation is given by

$$\sigma_{total} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \qquad (3.77)$$

where

σ_x is the standard deviation for x-axis

σ_y is the standard deviation for y-axis

σ_z is the standard deviation for z-axis

The simulation gives the following parameters as output

- Standard deviation for each axis as follows (in meters)

    Std_Dev_Data =

        29.2089 (x axis)

        30.0181 (y axis)

        80.2068 (z axis)

- Total standard deviation as follows (in meters)

    Tot_Std_Dev =

        90.4842

- Plot of the locations of the receivers are shown in Figure 27 and an exaggerated confidence ellipsoid is added in Figure 28.
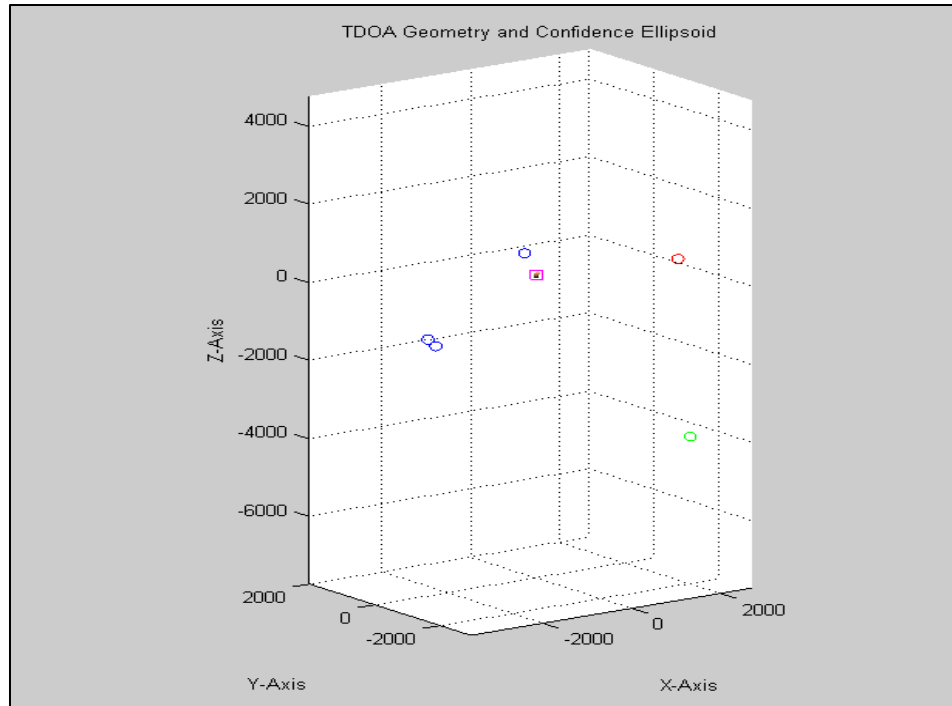
Figure 27.   TDOA Geometry in 3D

The exaggerated confidence ellipsoid as in Figure 28 is the diagonalized confidence ellipsoid with axes exaggerated by a factor of 1000 for visibility and centered on the estimated position.
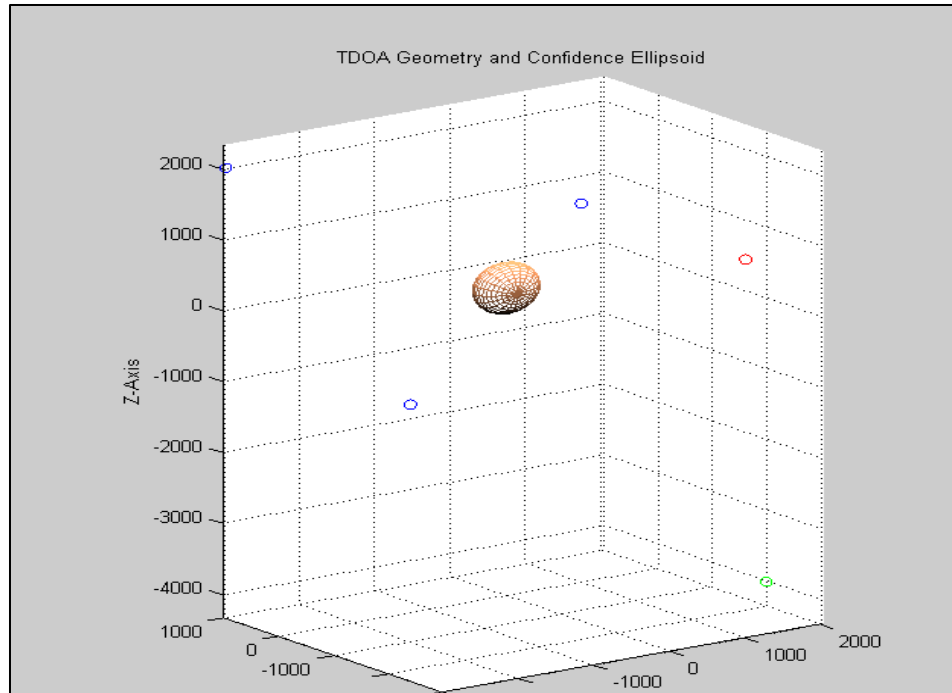
Figure 28.   TDOA Geometry and Exaggerated Confidence Ellipsoid

The simulation is run for the following analysis

- The effect of the distance on the accuracy,
- The effect of the noise on the accuracy,
- Comparison between moving receivers and the stationary receivers,
- The optimum distribution of the receivers for better accuracy.

1.   The Effect of the Distance on the Accuracy

The developed simulation is in Appendix C.

The effect of the distance on the accuracy is analyzed in different ways; distance change in one axis, distance changes in two axes, and distance changes in all three axes. Figure 29 shows the effect of the distance in one axis, Figure 30 shows the effect of the distance in two axes and Figure 31 shows the effect of the distances in three axes.
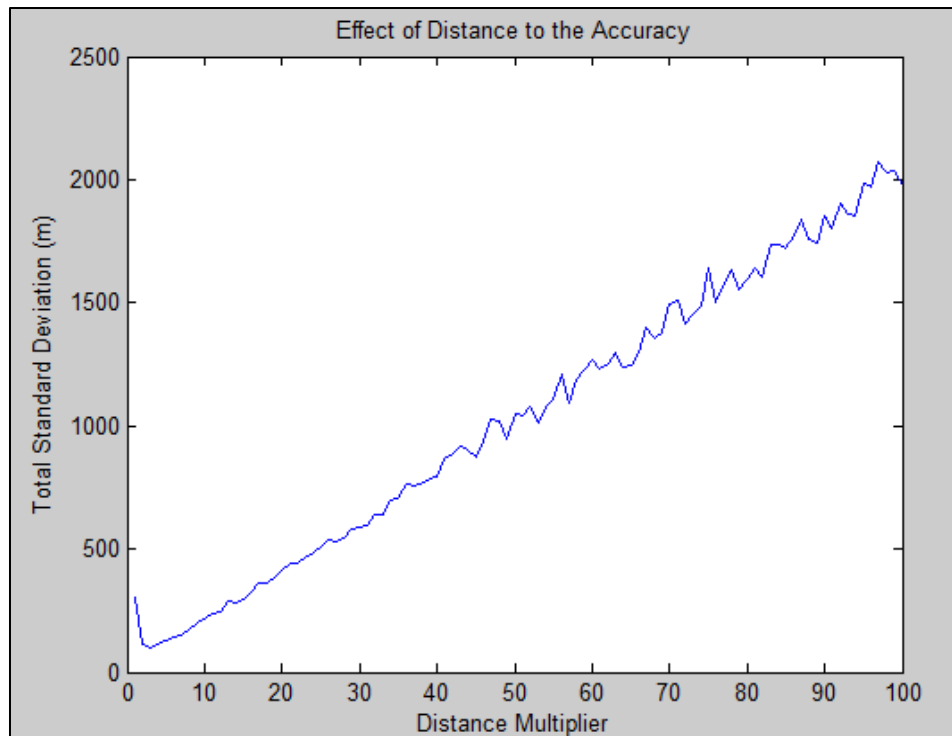
Figure 29.   Effect of Distance to Accuracy for x-axis
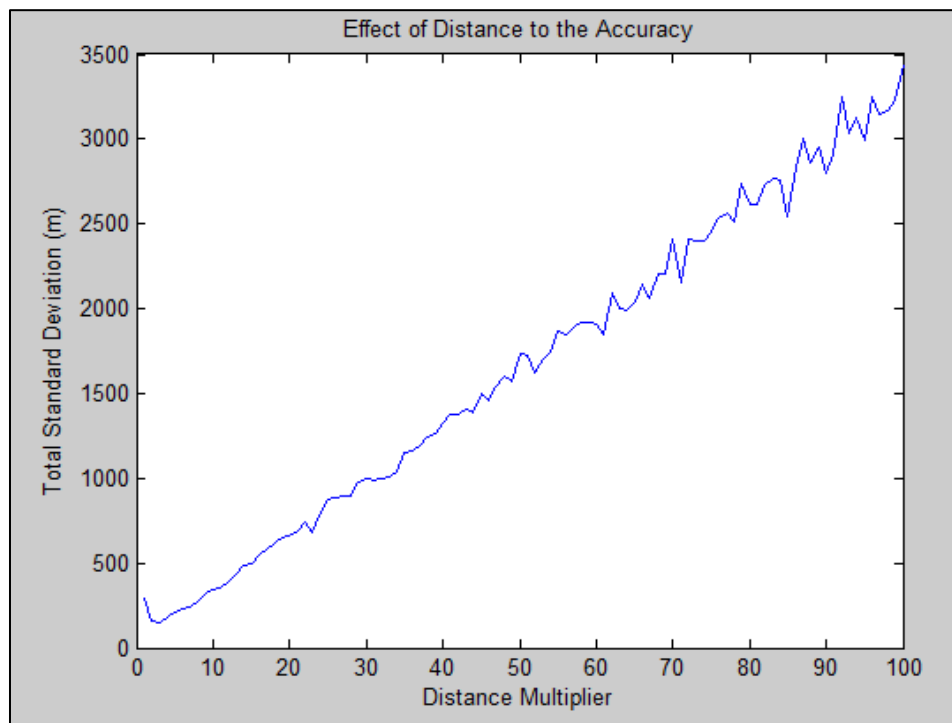
Figure 30.   Effect of Distance to Accuracy for two Axes (x and y axis)
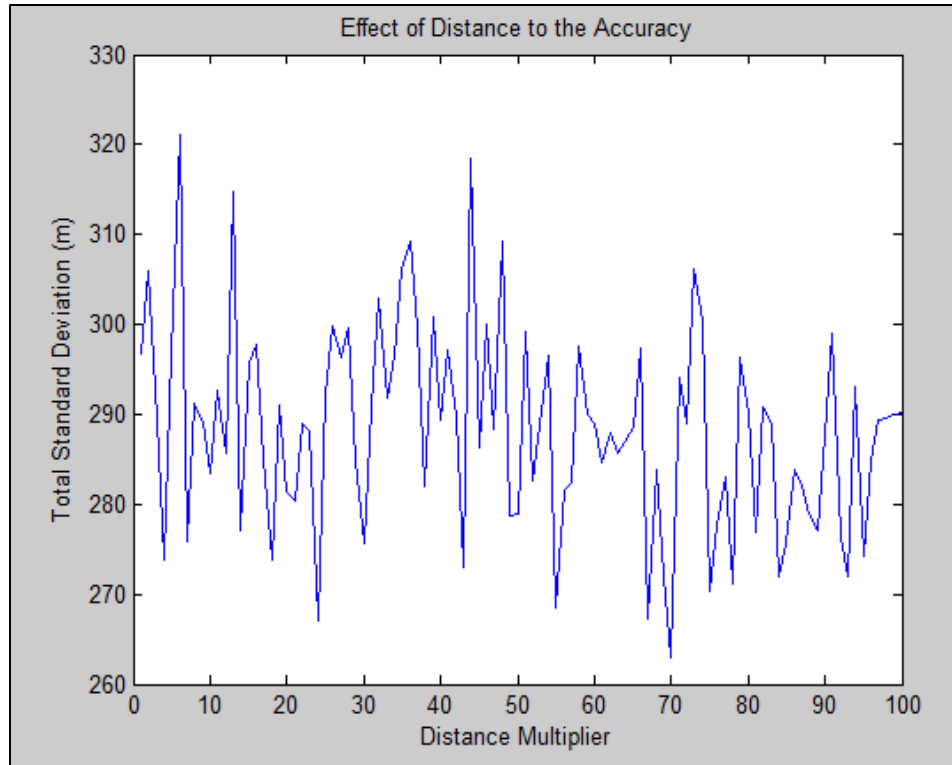
Figure 31.   Effect of Distance to Accuracy for Three Axes

The distance affect the total standard deviation linearly since the distance change happens in one or two axes since these kind of changes result in a change in the shape of the geometry of the receiver location. The distance change in two axes affect the total standard deviation more rapidly compared to the distance change in one axis.

On the other hand, total standard deviation is not effected linearly from distance change in three axes because changes in three axes result in changing the scale of the cluster, nothing more. The average total standard deviation in three axes distance change is around 300 meters for this configuration of random geometry.

2.   The Effect of the Noise on the Accuracy

Noise affects the system and the simulation as

$$TDOA = t + e_N \qquad\qquad (3.78)$$

where t is the time of arrival and e_N is the noise error.

The noise error changes for each receiver pair. For this simulation, the noise distribution is considered as a normal distribution with a standard deviation σ_x and zero mean for noise error.

The developed simulation for stationary receivers is in Appendix D.

The simulation is run for various standard deviations to understand the effect of the noise on the system. Figure 32 shows the effect of the noise on the accuracy.
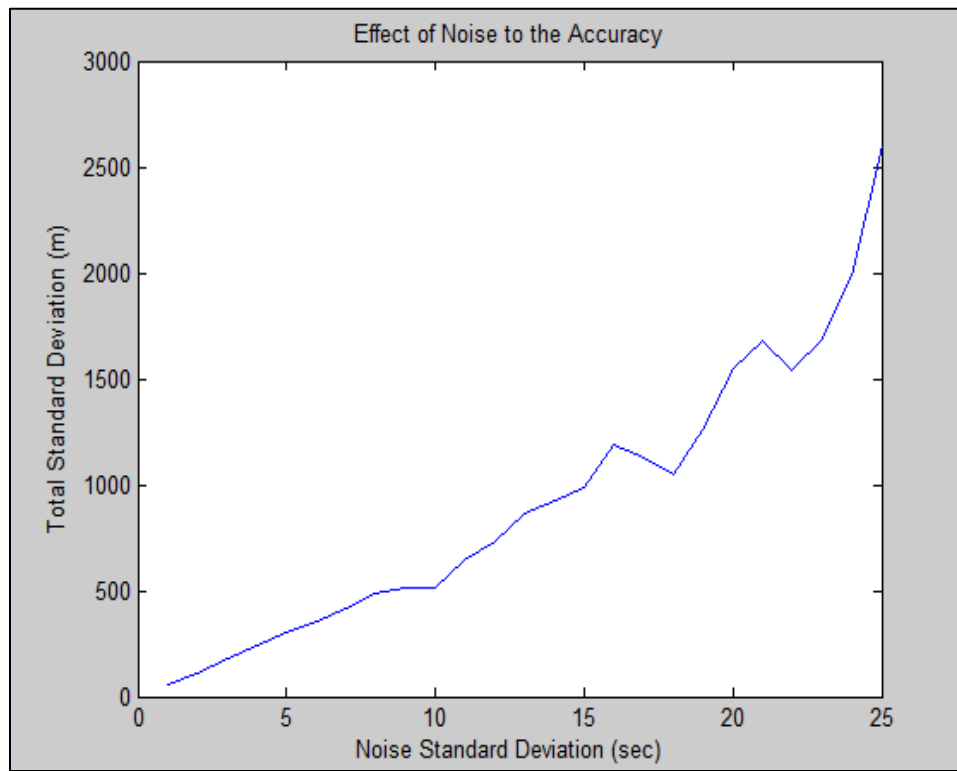
Figure 32.   The Effect of Noise on Accuracy

From Figure 32, it can be seen that the accuracy in terms of total standard deviation changes linearly with respect to noise standard deviation. This is a

good method of increasing the Signal to Noise Ratio (SNR) to keep the accuracy high.

This is consistent with the theoretical result that the diagonalized variances of the estimated position are proportional to the measured TDOA variances.
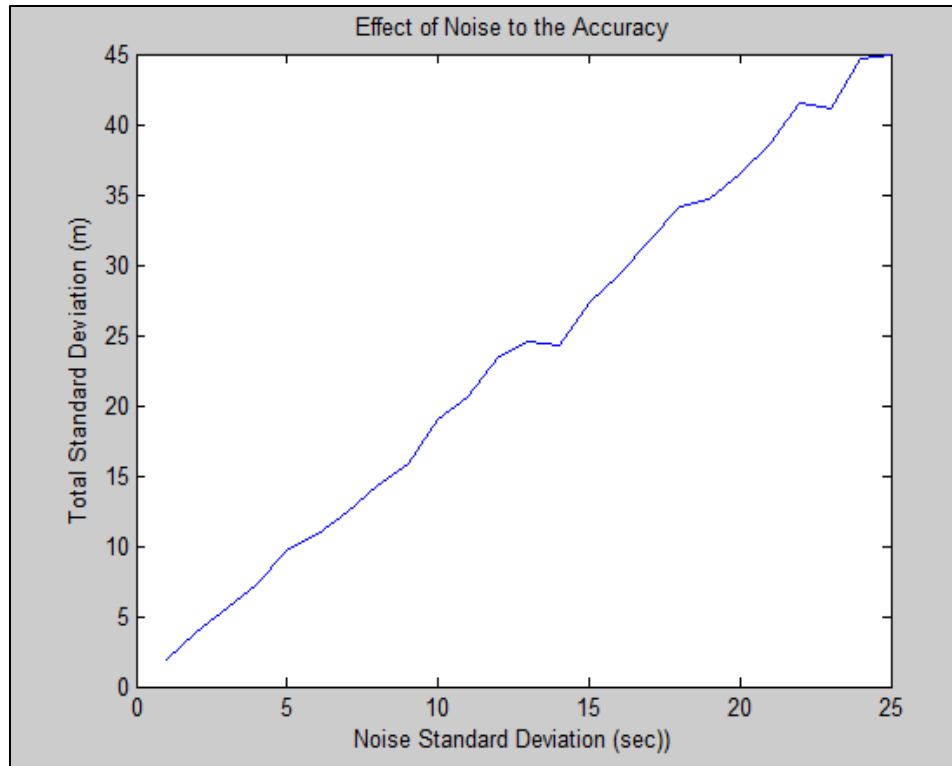
Figure 33.   The Effect of Noise to Accuracy with Moving UAV

From Figure 33, noise effect with moving UAV can affect the accuracy just like the way it affects without the UAV, but total standard deviation for estimated emitter location with UAV is smaller than the one without the UAV.

3.   Comparison Between Moving Receivers to the Stationary Receivers

Ground forces can use UAVs instead of stationary receivers even though the use of stationary receivers is more probable. The simulation is developed to show the effect of moving UAVs to the accuracy in terms of total standard deviation.

The developed simulation is in Appendix F for both one and two flying UAVs.

The simulation is run once for five random stationary receivers, four random stationary receivers plus a moving UAV and three random stationary receivers plus two moving UAVs. The simulation is run for random stationary receivers first, then the same procedure is followed for four random stationary receivers and a moving UAV. The same procedure is followed for the third scenario where there are three random stationary receivers plus two moving UAVs. For each set of receiver positions, the emitter is placed at (0,0,0) and 500 TDOAs with random noise are generated.

The results are analyzed in terms of the standard deviation for each axis and total standard deviation, which is a combination of standard deviations of all axes.

Figure 34 shows the random locations of five stationary receivers which try to locate the emitter.
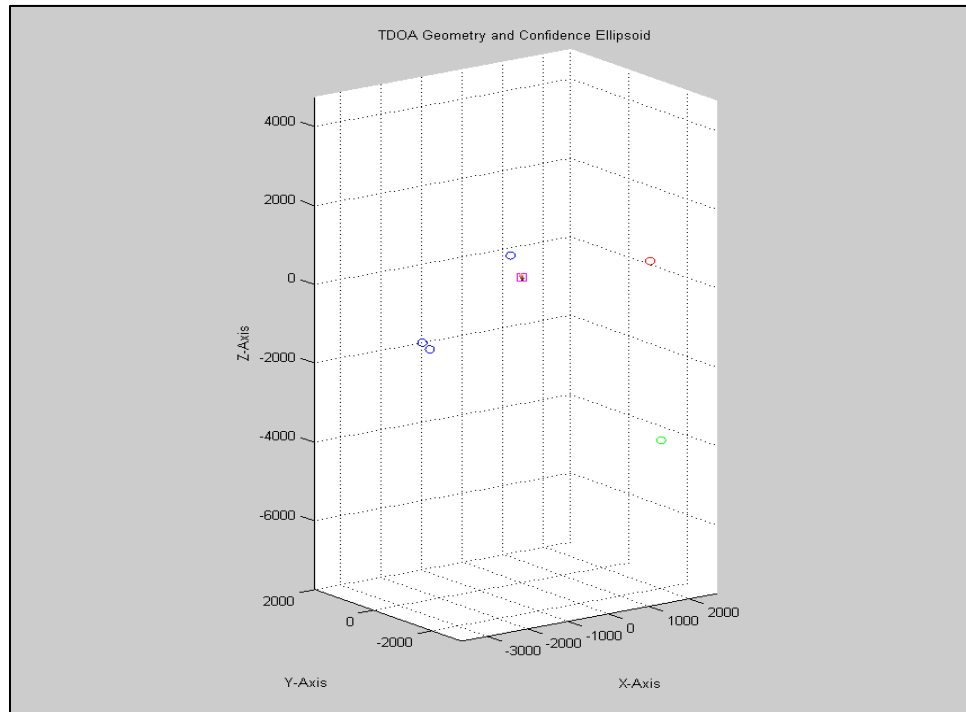
Figure 34.   Random Locations of Stationary Receivers

Figure 35 shows the magnified confidence ellipsoid for five stationary receivers.
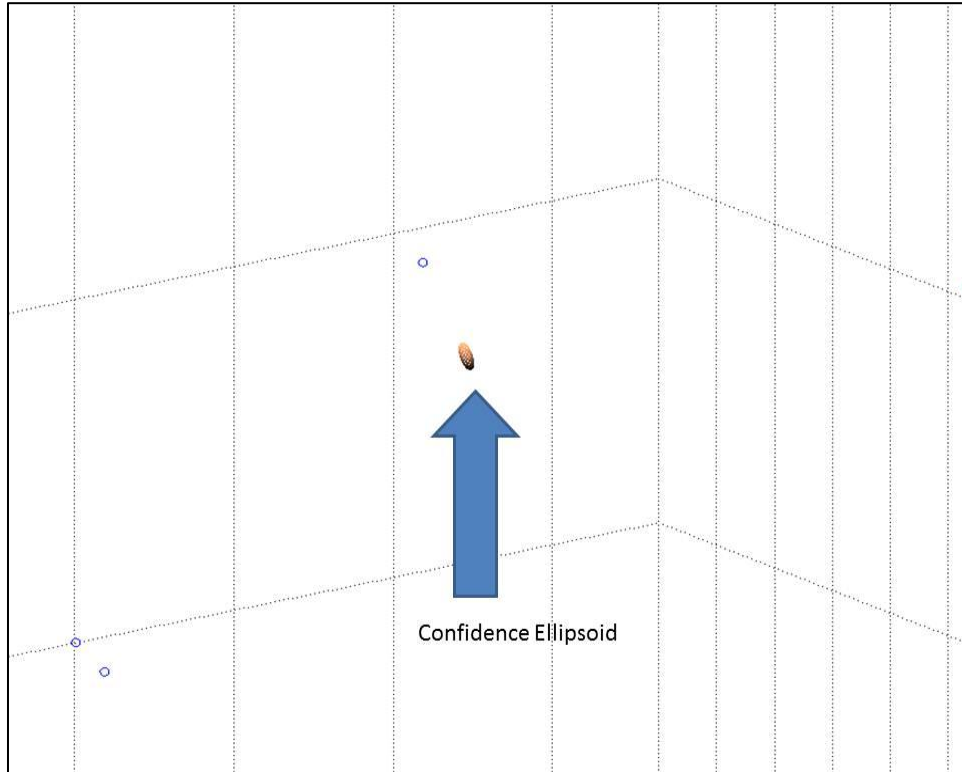
Figure 35.   Magnified Confidence Ellipsoid for Random Stationary Receivers

Standard deviations for each axis for five random stationary receivers are:

σ_x = 30.3517 m

σ_y = 32.7943 m

σ_z = 77.5429 m

Total standard deviation for five random stationary receivers is:

σ_T = 89.4964 m

Figure 36 shows the random locations of four receivers and a moving UAV which try to locate the emitter.

The UAV flies over the operation area at a constant altitude for that scenario. 20 points of the route of the UAV are used for experiments. 500 experiments are done at every point of the UAV. Total standard deviation is calculated for every point of the UAV. The results are collected in a matrix and plotted as in Figure 37.
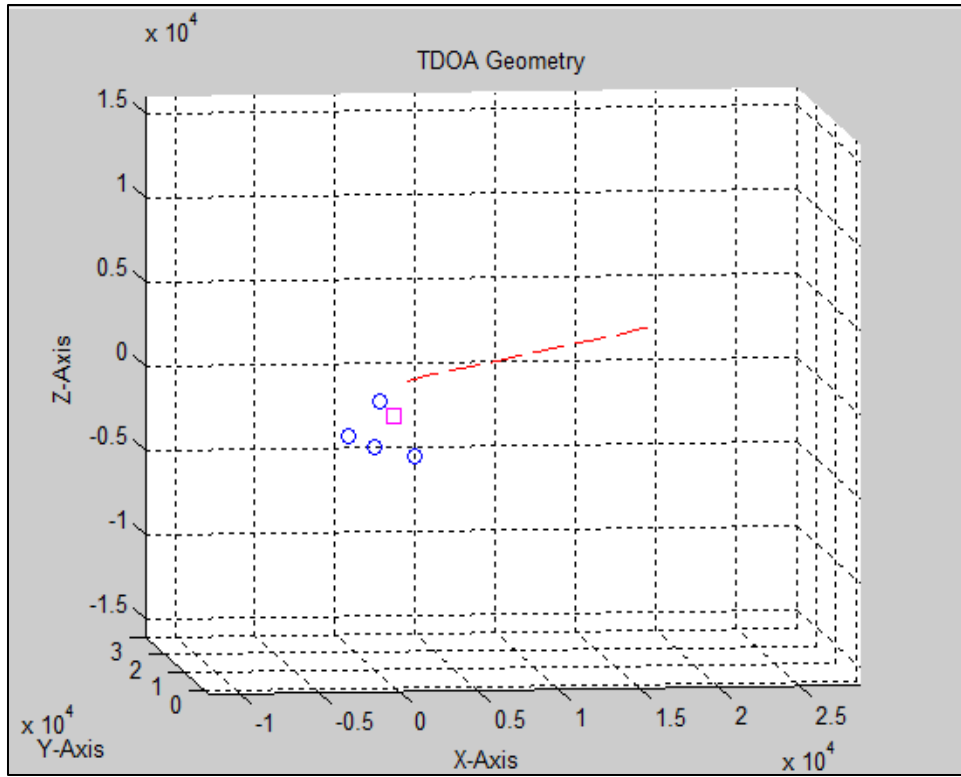

Figure 36.   Random Locations of Four Stationary Receivers and One Moving UAV
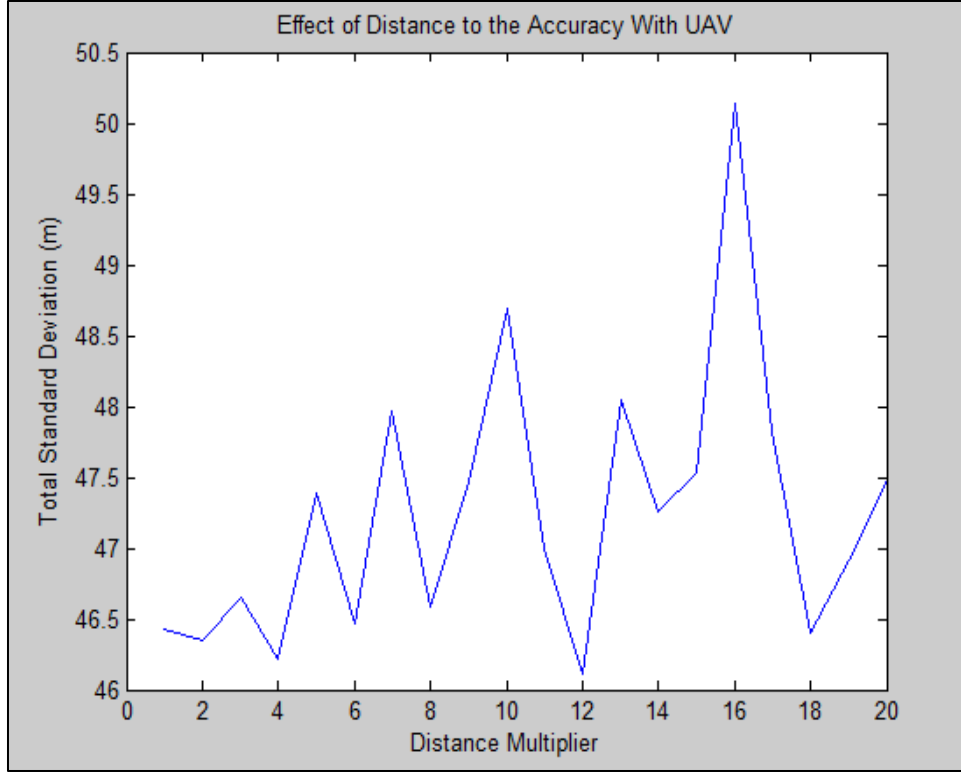
Figure 37.   Total Standard Deviation Change for Four Random Stationary Receivers and a Moving UAV

The confidence ellipsoid for each experiment is not plotted but the numeric values of standard deviation for each axes can be found in the proposed simulation code.

Figure 38 shows the random locations of three stationary receivers and two moving UAV, which try to locate the emitter.

The UAVs fly over the operation area at a constant altitude for that scenario. 20 points of the routes of each UAV are used for experiments. 500 experiments are done at every point pair of the UAVs. Total standard deviation is calculated for every point pair of the UAVs. The results are collected in a matrix and plotted as in Figure 39.
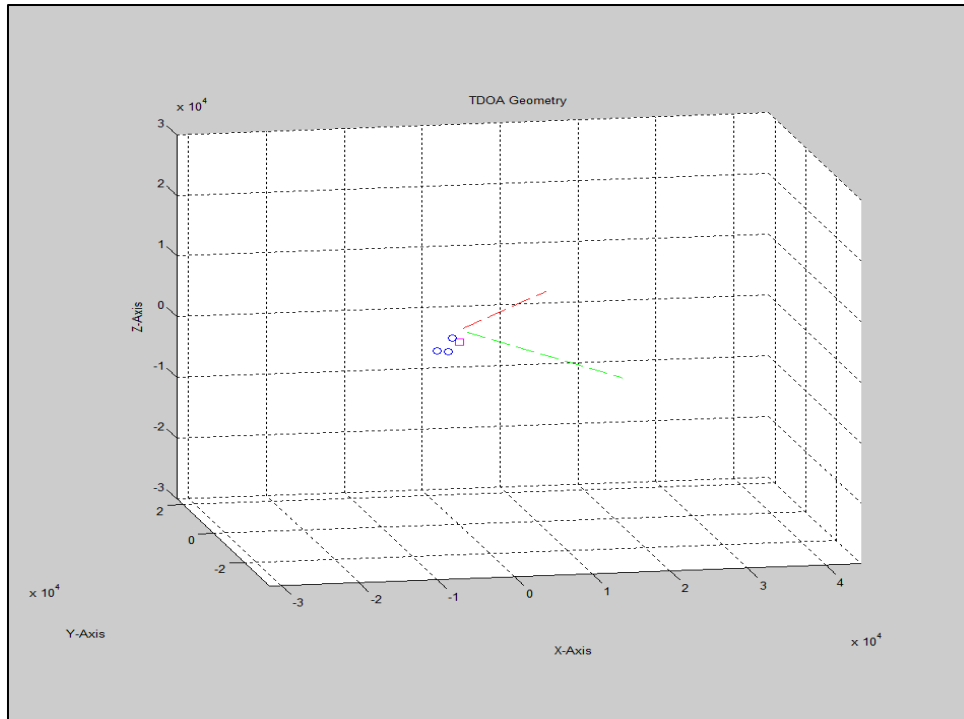
Figure 38.   Random Locations of Three Stationary Receivers and Two Moving UAVs
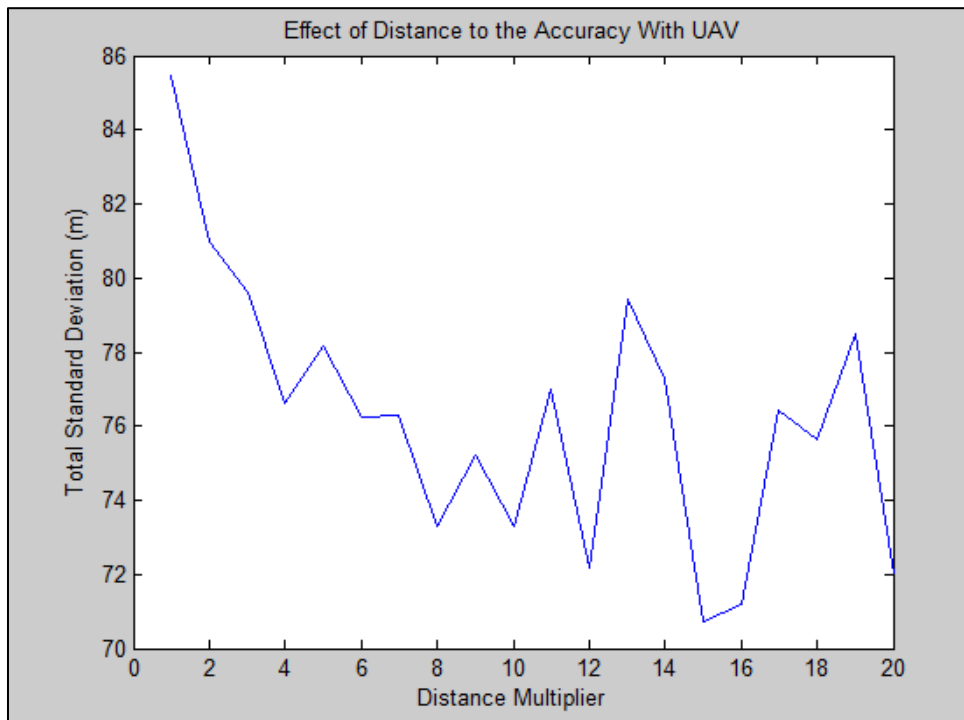
Figure 39.   Total Standard Deviation Change for Three Random Stationary Receivers
and two Moving UAVs

The confidence ellipsoid for each experiment is not plotted but the numeric values of standard deviation for each axes can be found in the proposed code.

It is clear from these datum that four random stationary receivers and a flying UAV has the smaller standard deviation for each axis, and they also have smaller total standard deviation compared to the other scenarios.

4.   The Optimum Distribution of the Receivers for Better Accuracy.

Four receivers can be positioned around the emitter using different geometric patterns, such as straight, trapezoidal, parallelogram, inverted triangle, Y shaped, lozenge, square, and rectangle as explained in Yan-Ping's article. 3D curves of position error are showed in Figure 40 and Figure 41.
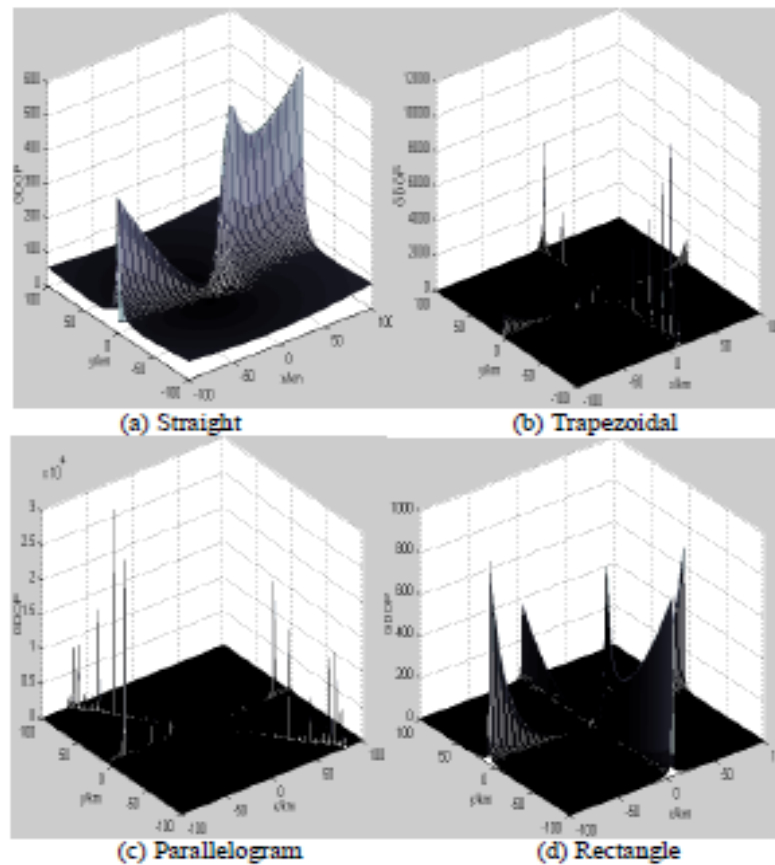
Figure 40.   3D Curves of Position Error for Straight (a), Trapezoidal (b), Parallelogram (c) and Rectangle (d)
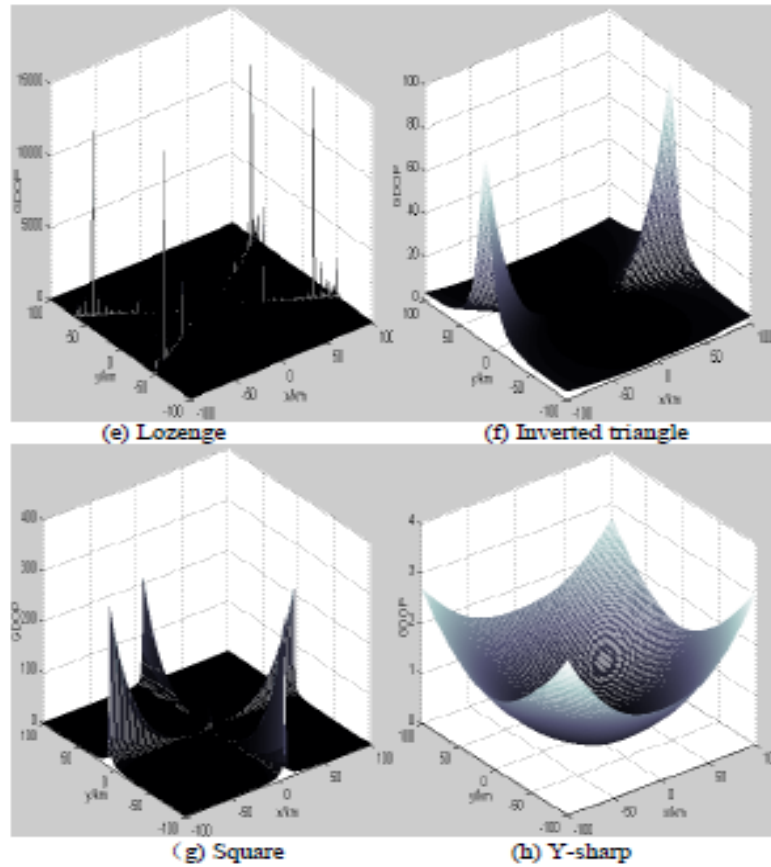
Figure 41.   3D Curves of Position Error for Lozenge (e), Inverted Triangle (f), Square (g), Y-Shaped (h)

Coverage areas are shown in Table 2.

Scene	Coverage value
Straight	10.2%
Trapezoidal	29.4%
Parallelogram	40.5%
Rectangle	40.9%
Lozenge	54.9%
Inverted triangle	58.6%
Square	61.6%
Y-sharp	88.6%

Table 2.   Coverage Values for Different Type of Receiver Distributions (From Yan-Ping et al, 2010)

It is clear that the Y-shaped receiver distribution is the best for coverage values of the desired reach area.

The developed simulation is in Appendix F.

The following position parameters are used in the simulation for eight different scenarios,

Scenario 1: Receivers are around the emitter (without altitude difference) as in Figure 42. The units are in kilometers (km).

X1 = 2;	Y1 = 0;	Z1 = 1.97;
X2 = 0.7;	Y2 = 2;	Z2 = 1.98;
X3 =-2;	Y3 = 1.5;	Z3 = 1.96;
X4 = -2;	Y4 = -1.5;	Z4 = 2;
X5 = 1;	Y5 = -1.5;	Z5 = 1.99;
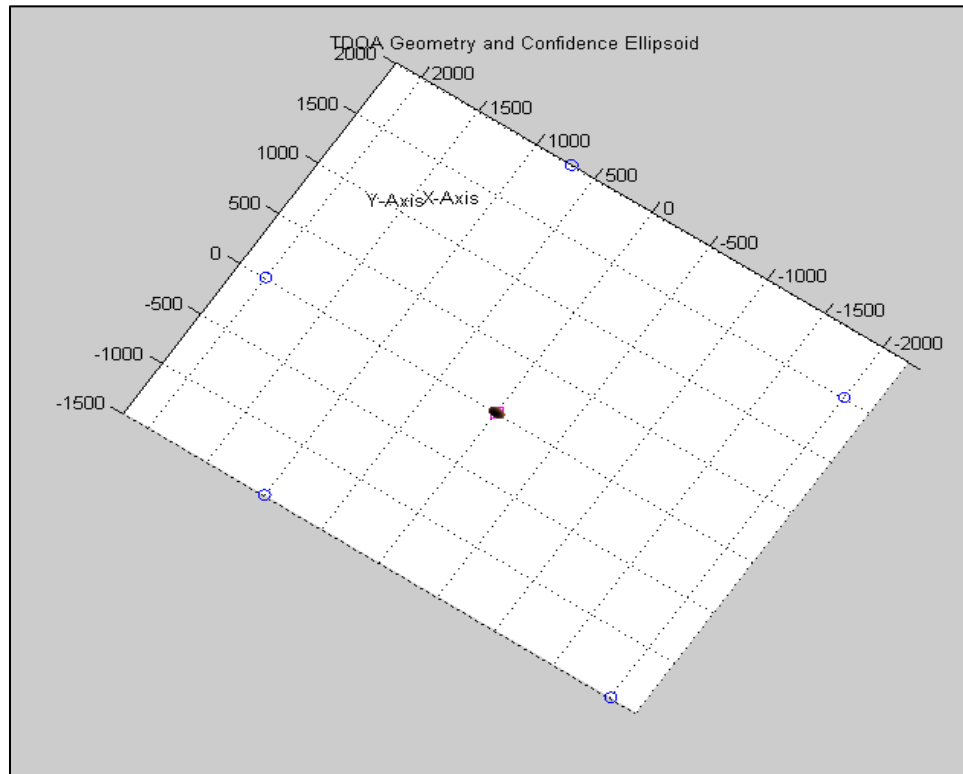
Figure 42.   TDOA Geometry for Scenario 1

Scenario 2: Receivers are on a straight line (without altitude difference) as in Figure 43. The units are in kilometers (km).

$$X1 = -2; \quad Y1 = -1; \quad Z1 = 0.99;$$
$$X2 = -1; \quad Y2 = 0; \quad Z2 = 1;$$
$$X3 = 0; \quad Y3 = 2; \quad Z3 = 0.98;$$
$$X4 = 1; \quad Y4 = 3; \quad Z4 = 0.97;$$
$$X5 = 1.5; \quad Y5 = 4; \quad Z5 = 1.1;$$
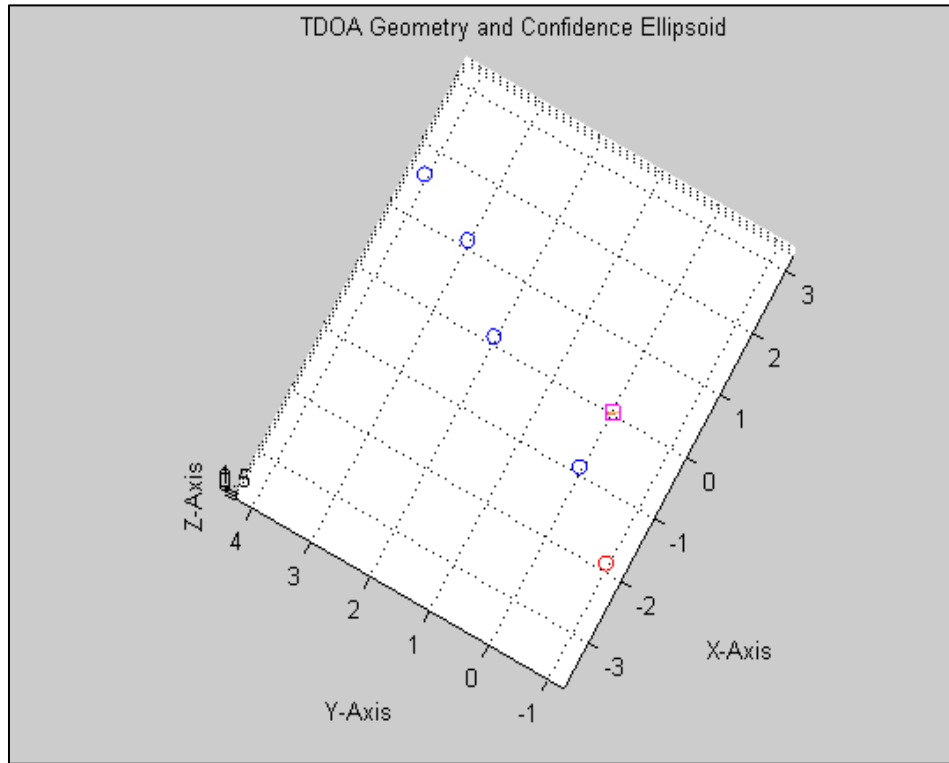
Figure 43.   TDOA Geometry for Scenario 2

Scenario 3: Receivers are around the emitter (with altitude difference) as in Figure 44 and Figure 45. The units are in kilometers (km).

$$X1 = 2; \quad Y1 = 0; \quad Z1 = 0;$$
$$X2 = 0.7; \quad Y2 = 2; \quad Z2 = 1;$$
$$X3 = -2; \quad Y3 = 1.5; \quad Z3 = -1;$$
$$X4 = -2; \quad Y4 = -1.5; \quad Z4 = 2;$$
$$X5 = 1; \quad Y5 = -1.5; \quad Z5 = -2;$$
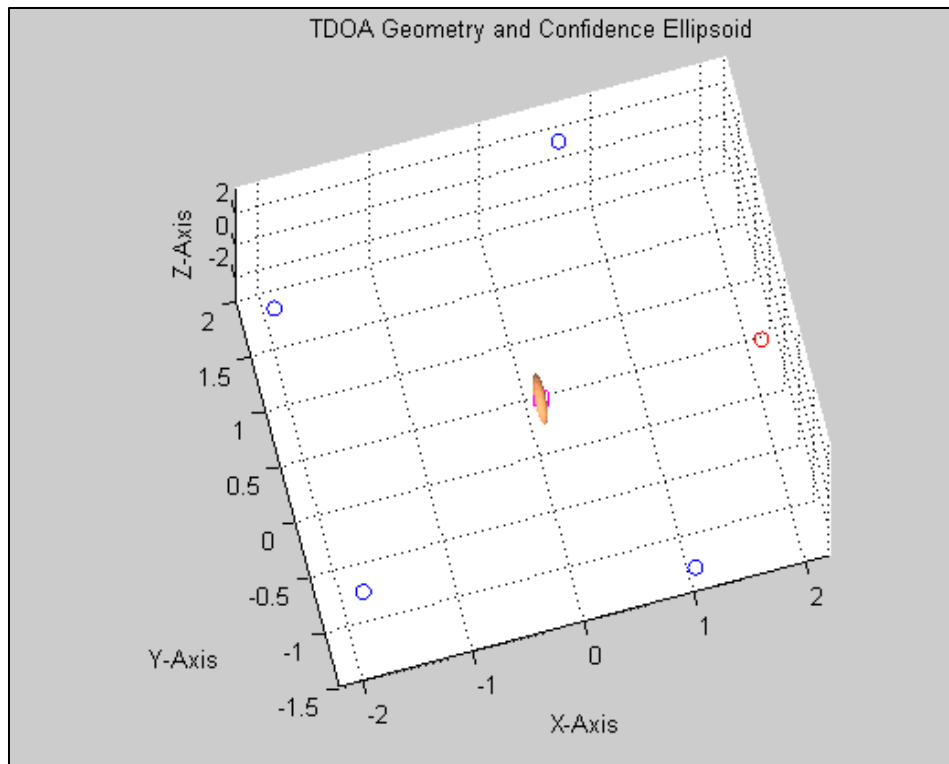
Figure 44.   TDOA Geometry for Scenario 3 (Top View)
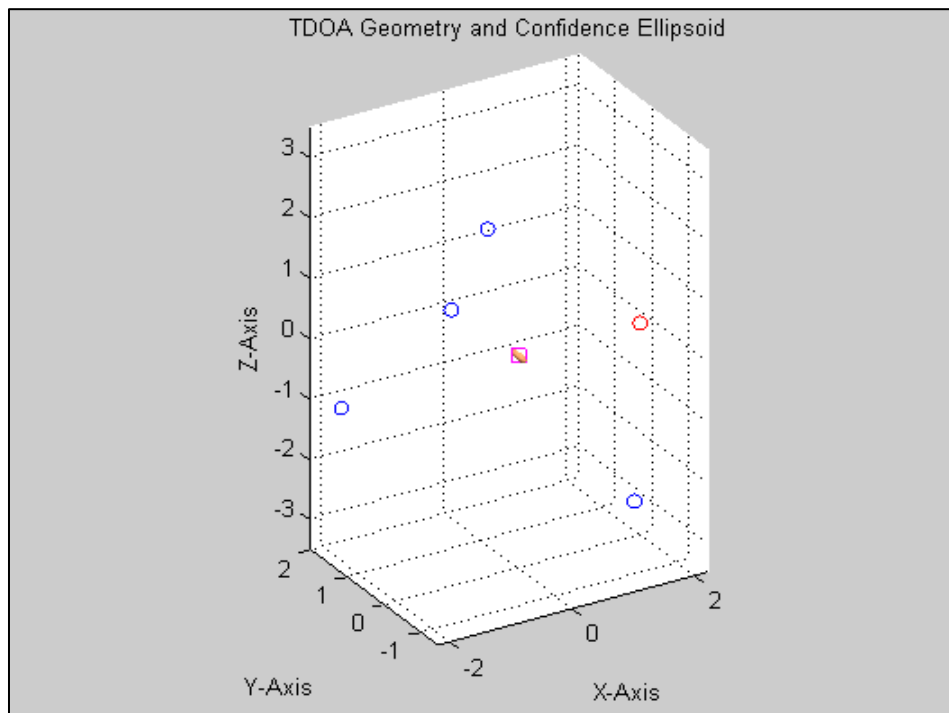

Figure 45.   TDOA Geometry for Scenario 3 (Side View)

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Scenario 4: Receivers are on a straight line (with altitude difference) as in Figure 46. The units are in kilometers (km).

$$X1 = -2; \quad Y1 = -1; \quad Z1 = 0;$$
$$X2 = -1; \quad Y2 = 0; \quad Z2 = 1;$$
$$X3 = 0; \quad Y3 = 2; \quad Z3 = 2;$$
$$X4 = 1; \quad Y4 = 3; \quad Z4 = 3;$$
$$X5 = 1.5; \quad Y5 = 4; \quad Z5 = 4;$$
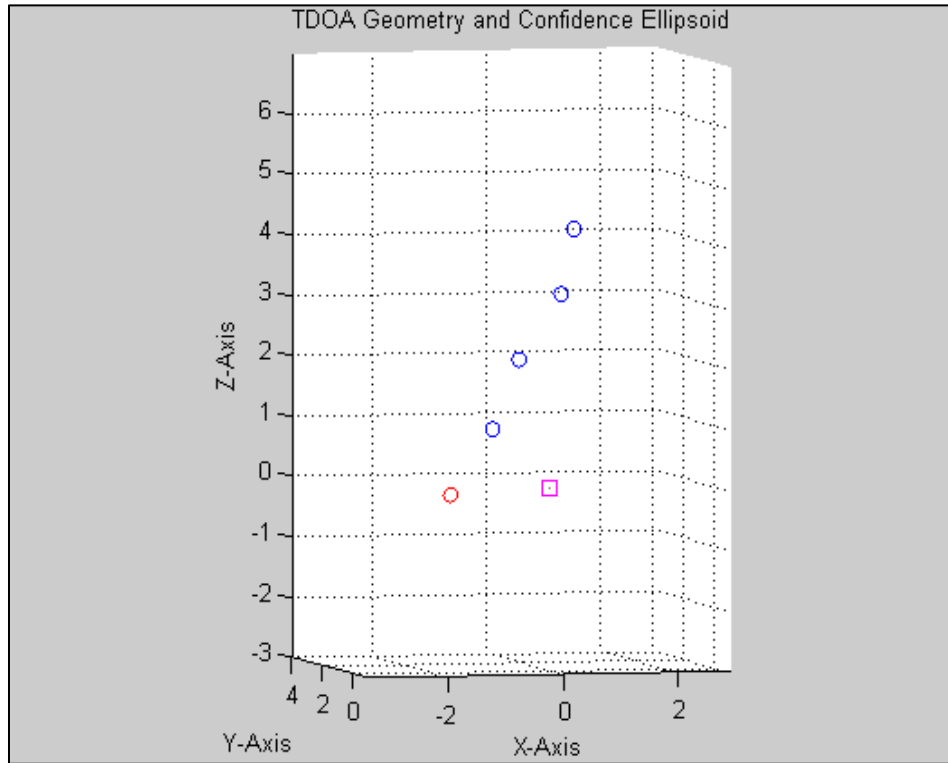
Figure 46.   TDOA Geometry for Scenario 4

Scenario 5: Receivers are in a trapezoidal formation as in Figure 47. The units are in kilometers (km).

$$X1 = 1; \quad Y1 = 0; \quad Z1 = 0;$$
$$X2 = 2; \quad Y2 = 2; \quad Z2 = -0.1;$$
$$X3 = 3; \quad Y3 = 2; \quad Z3 = -0.1;$$
$$X4 = 4; \quad Y4 = 2; \quad Z4 = -0.1;$$
$$X5 = 5; \quad Y5 = 0; \quad Z5 = -0.1;$$

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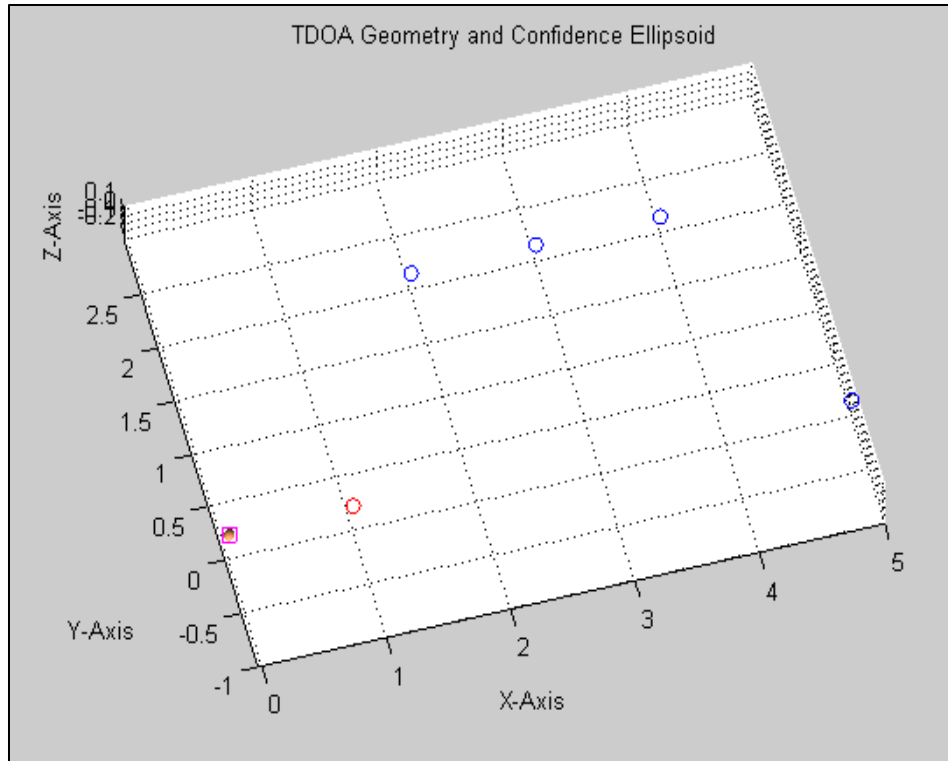
Figure 47.   TDOA Geometry for Scenario 5

Scenario 6: Receivers are in a parallelogram formation as in Figure 48. The units are in kilometers (km).

$$X1 = -1; \qquad Y1 = 0; \qquad Z1 = 0;$$
$$X2 = -2; \qquad Y2 = 2; \qquad Z2 = -0.1;$$
$$X3 = 0; \qquad Y3 = 2; \qquad Z3 = -0.1;$$
$$X4 = -2; \qquad Y4 = 0; \qquad Z4 = -0.1;$$
$$X5 = -3; \qquad Y5 = 0; \qquad Z5 = -0.1;$$
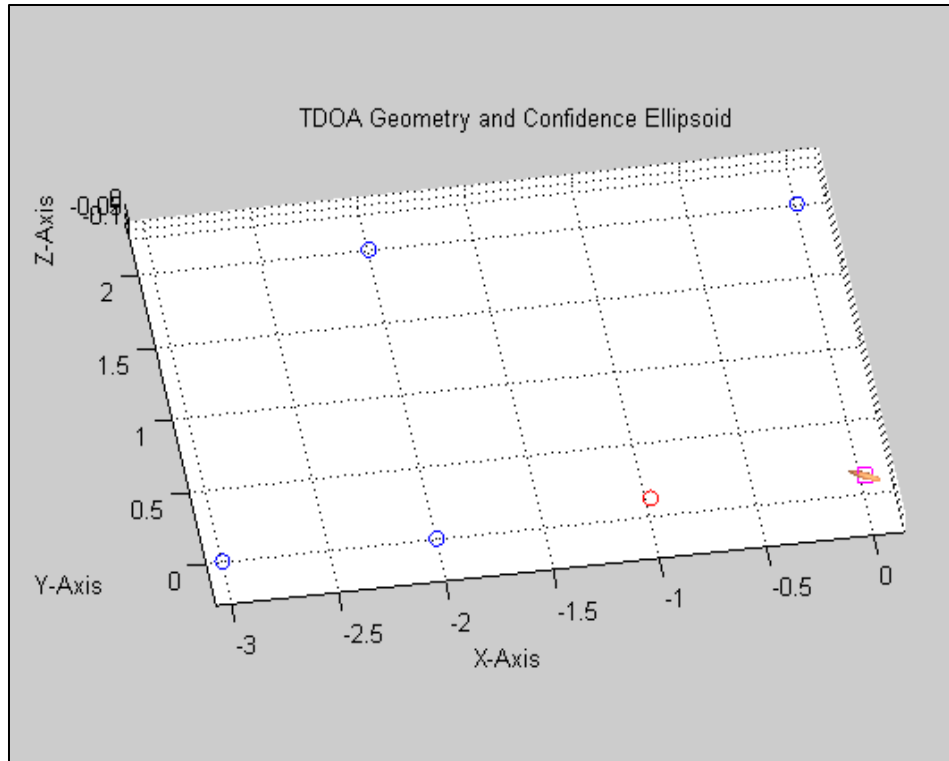
Figure 48.　TDOA Geometry for Scenario 6

Scenario 7: Receivers are in a lozenge formation as in Figure 49. The units are in kilometers (km).

$$X1 = 8; \qquad Y1 = 15; \qquad Z1 = -0.12;$$
$$X2 = -13; \qquad Y2 = 12.5; \qquad Z2 = -0.13;$$
$$X3 = 0; \qquad Y3 = 18; \qquad Z3 = -0.14;$$
$$X4 = 13; \qquad Y4 = 12.5; \qquad Z4 = -0.11;$$
$$X5 = 0; \qquad Y5 = 8; \qquad Z5 = -0.1;$$
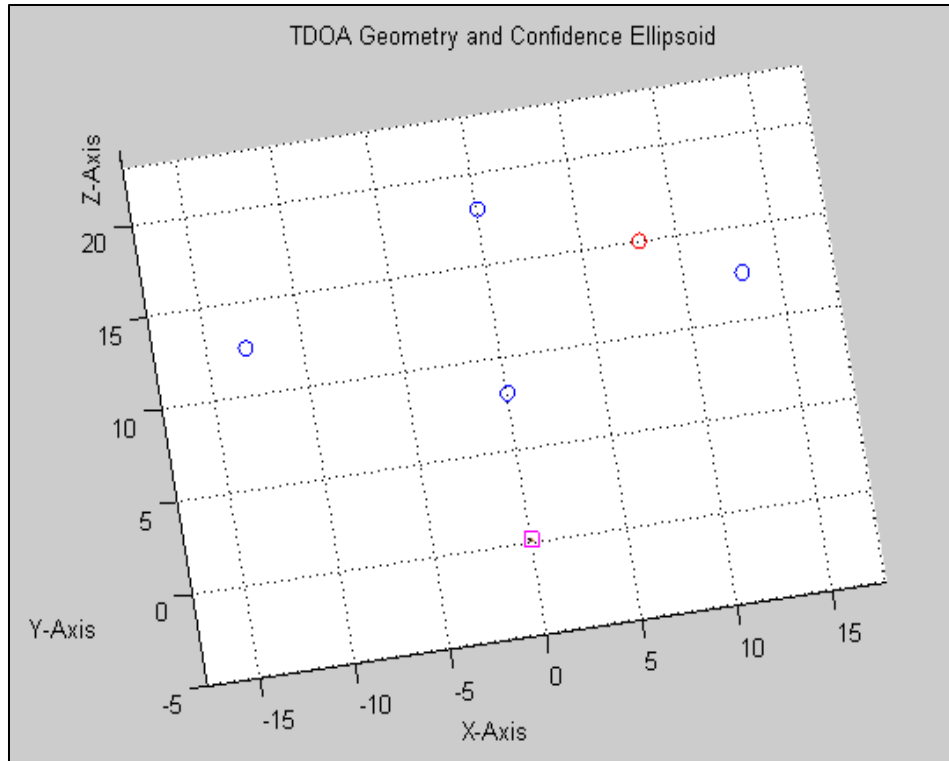
Figure 49.   TDOA Geometry for Scenario 7

Scenario 8: Receivers are in an inverted triangle formation as in Figure 50. The units are in kilometers (km).

$$X1 = 2; \qquad Y1 = 0; \qquad Z1 = 0;$$
$$X2 = 4; \qquad Y2 = 2; \qquad Z2 = -0.1;$$
$$X3 = 6; \qquad Y3 = -2; \qquad Z3 = -0.1;$$
$$X4 = 8; \qquad Y4 = 4; \qquad Z4 = -0.1;$$
$$X5 = 10; \qquad Y5 = -4; \qquad Z5 = -0.1;$$
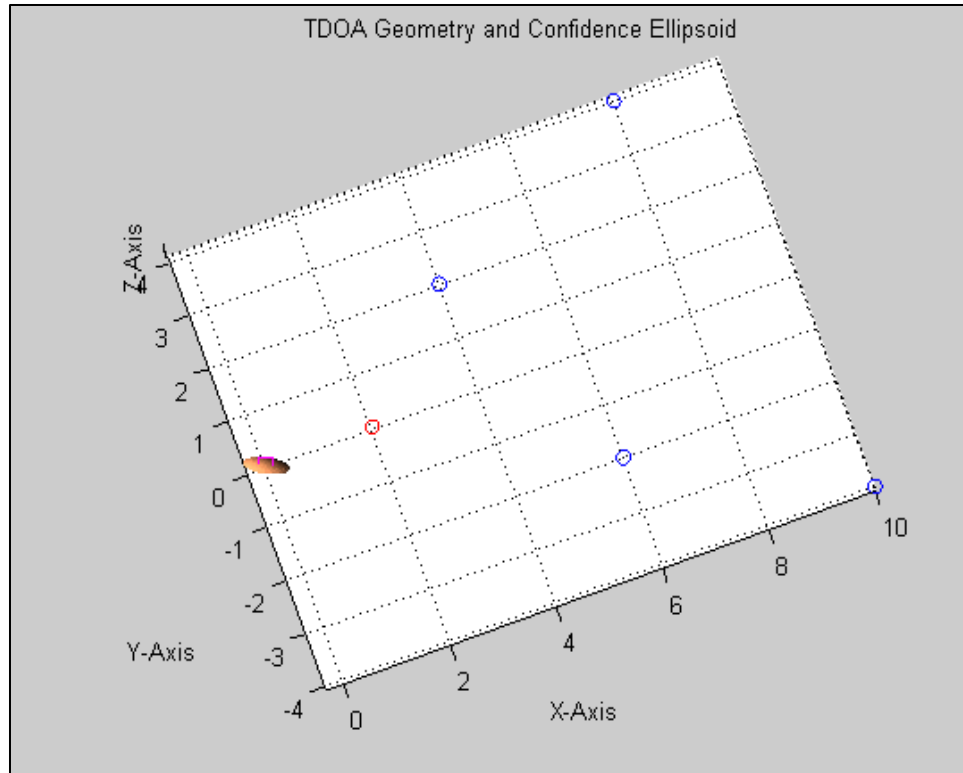
Figure 50.   TDOA Geometry for Scenario 8

Table 3 shows the total standard deviations for all eight scenarios.

Scenario #	Total Standard Deviation
1	3.9003 km
2	1.2128 km
3	67.2223 m
4	751.0645 m
5	9.5230 km
6	1.4135 km
7	1.2795 km
8	3.7662 km

Table 3.   Estimated Emitter Location and Total Standard Deviation for Position
Scenarios

It is clear from Table 3 that scenario 3 has the best two total standard deviations. This type of receiver positioning should be used to get the best accuracy if the system is using the proposed close-form geolocation technique.

V. CONCLUSION AND RECOMMENDATIONS

A.   CONCLUSION

Electronic warfare plays a dominant role in today's technological military world. Electronic warfare support is one of the core elements of EW, to include direction finding and geolocation.

Passive geolocation of an emitter, requiring no emission of energy by the receiving and locating devices, plays an important role in both military and civilian applications. Passive geolocation can locate targets by measuring their electromagnetic emissions.

The greatest internal problem for the Turkish military at present is the terrorist groups operating within the southeast part of Turkey. Military forces have been conducting operations against them for nearly 30 years. The operations occur in two phases, find and destroy. Finding the location of the terrorists in the mountainous areas of Turkey is crucial. Civilian and military intelligence has been used at the strategic level to locate them, but without operational and tactical location systems success is fleeting at best.

One approach that uses multiple receivers is TDOA, which is the main concern of this thesis. For this technique, the time of arrival of the received signal at different receivers is measured; the measurements of the times of arrival are then used to find the time difference of arrival. These differences are used to determine the location of the emitter.

Finding the time difference of arrival for radar signals is relatively easy, compared to CW signals. Because radar signals contain pulses, and these pulses are almost square (ideally), they have a defined beginning and an end. These beginnings and endings provide a time mark for time difference of arrival. These time marks are used for measuring the times of arrival; subtracting times of arrival results in a time difference of arrival. For a CW signal, there are no time marks. They do not have a specific beginning or end, which makes time

difference of arrival more difficult to calculate. A cross-correlation technique has to be used for CW signals to find the time difference of arrival.

There is quite a bit of literature discussing many proposed methods to find the location of the emitter using the measured time difference of arrival. Many of the techniques require range data. The technique discussed in this thesis, Ezzat's technique, does not. His technique is a closed-form geolocation technique.

The areas where the operations take place are very mountainous. Because of this reason, it is necessary to use a 3D TDOA technique.

This thesis provided a brief discussion on the fundamentals of TDOA and a closed-form usage of TDOA. A simulation model was developed in order to observe the performance of the proposed close-from technique in different scenarios. The effects of the distance, noise and distribution of the receivers to accuracy were analyzed. A comparison between a system of stationary receivers and a system of combinations of stationary and moving receivers was done to understand the best combination of receivers for best accuracy. For each combination of receivers, 500 simulations with added independent noise were run to acquire an estimate of the emitter position and to obtain diagonalized covariance values.

The developed simulation was based on Ezzat's closed-form technique. His technique was repeated many times to understand its robust nature.

Distance affects accuracy in a linear manner, as explained in Chapter 4 Section C. Being close to the emitter gives the best accuracy. The closer to the emitter, the better the accuracy.

Noise also has a linear effect on accuracy, as explained in Chapter 4 Section C. When the standard deviation of the noise increases, the standard deviation of the position estimate increases proportionally, that results a reduction in the accuracy. Since the noise considered in the simulation is background noise and cannot be removed from the system, the best way to

overcome this problem is to use a receiver with a better sensitivity so as to achieve a higher signal to noise ratio.

The system should include at least four stationary and one flying UAV as the fifth receiver for better accuracy as explained in Chapter 4 Section C. It is clear from the simulation that having a flying UAV with stationary ground receivers has a significant effect on the accuracy. One other advantage of having a UAV is not losing the line of sight with the signal of interest. It is difficult to have line of sight at all times, especially in high clutter areas that the Turkish military forces operate. UAVs can solve this problem. They are also faster and more mobile than ground receivers, which make them preferable for better coverage. UAVs also offer the advantage of adding the FDOA technique to the calculation of the emitter/enemy location, a discussion that was beyond the interest area of this thesis, but is relevant to the objective of locating emitters.

The two best distributions of receivers are shown in Scenarios 3 and 4, as explained in Chapter 4 Section C. One solution is to distribute the receivers around the emitter with altitude differences as in Figures 51 and 52. Figure 51 shows the geometry from the side view and Figure 52 shows the geometry from the top view. The other distribution is to have the receivers on a baseline with altitude differences as in Figure 53. The operational forces should try to locate the receivers as explained in one of these scenarios for best accuracy.
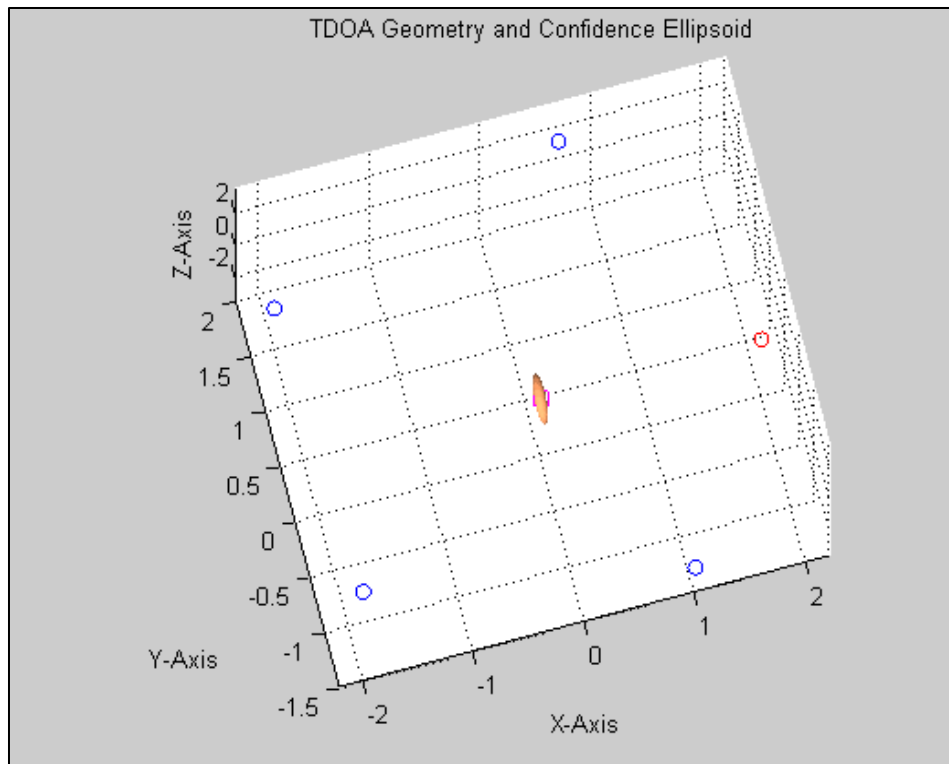
Figure 51.   TDOA Geometry of the Receivers around the Emitter (Top View)
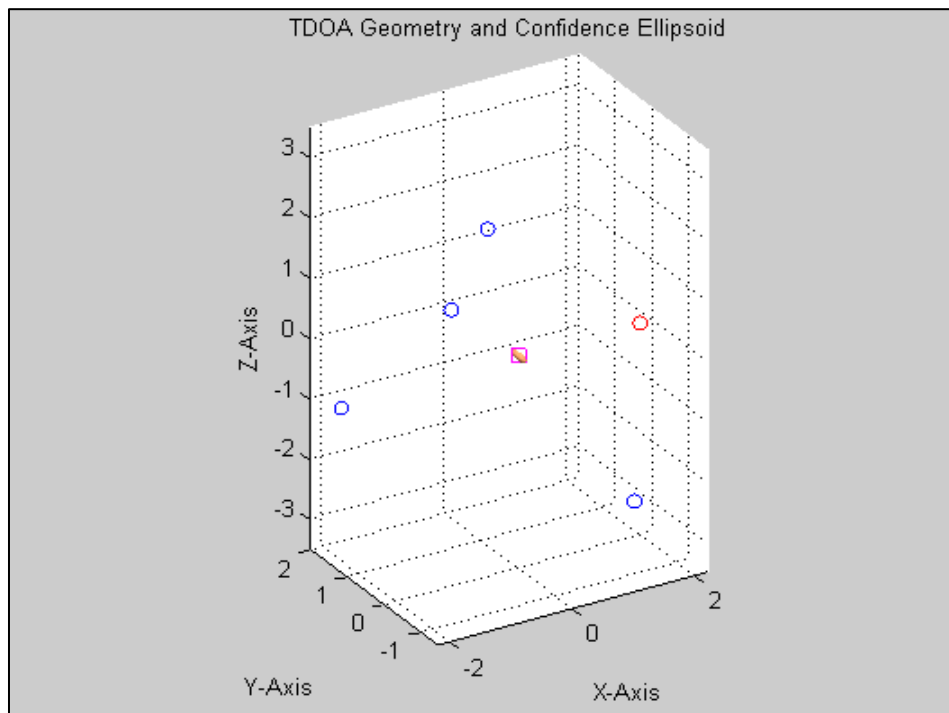

Figure 52.   TDOA Geometry of the Receivers around the Emitter (Side View)
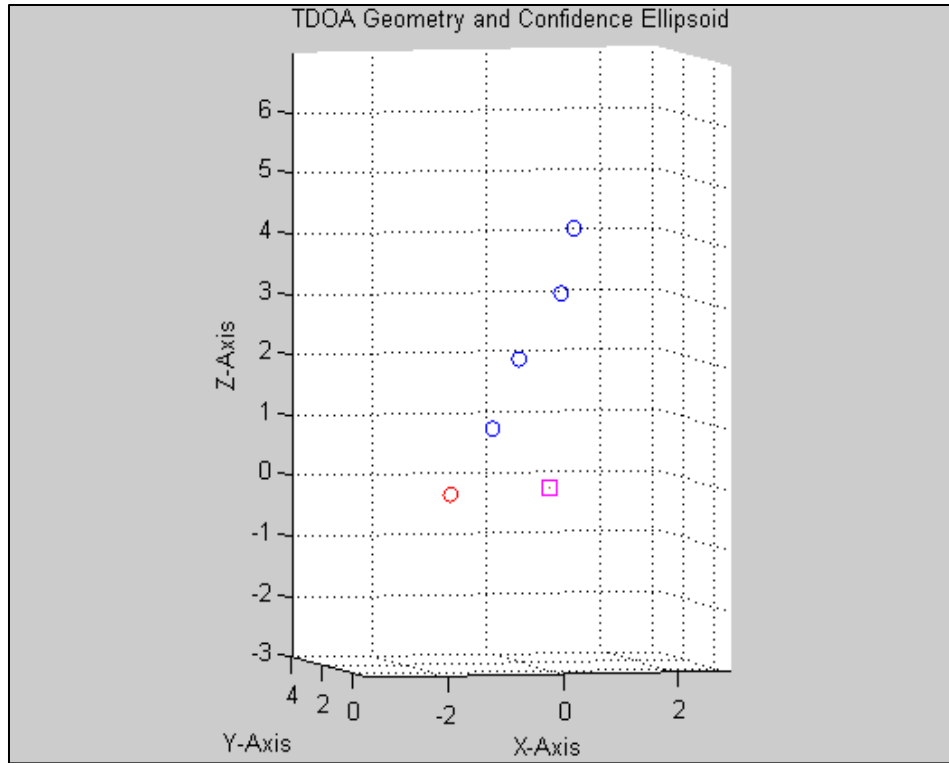
Figure 53.   TDOA Geometry of the Receivers on a Straight Line with Altitude
Difference

The main disadvantage of the proposed closed-form geolocation system is a requirement to use one more receiver than with some other TDOA techniques, for which it is only necessary to have four TDOA receivers.

The simulation enables the user to analyze the performance of the closed-form geolocation technique under desired conditions. It must be noted that the values obtained from the simulation may differ from real environment values, since many assumptions were made and certain values were kept constant in order to simplify the optimization process.

From the Turkish military perspective, since the developed model examines specific conditions, the model can be upgraded to an advanced level according to the needs and capabilities of the Turkish military and can be an example model for improved future studies.

B. RECOMMENDATIONS

EW will maintain a critical place on the battlefield. Almost all nations are spending money for EW research. Every nation tries to take part in the intelligence arena. All these facts increase the importance of operational tactics development for these technologies.

It is obvious that closed-form geolocation has certain advantages in specified areas. The research conducted under this thesis tried to examine these features and created a model in order to simulate the environment. The simulation model can be a starting point and can be improved in several ways for future studies.

Contrasting the proposed closed-form geolocation technique with the other forms of TDOA geolocation that use repeated measurements involving at least one moving receiver might be a good research subject to determine which TDOA geolocation technique to use for better accuracy.

Instead of making assumptions, using fixed values and omitting certain facts in order to reduce the complexity, allowing for more detailed studies. Multipath effects can be added for better understanding. Allowing for multipath may well produce better results in the presence of mountainous clutter. The academic studies that have been used as references in this thesis are the primary resources that can lead to future research and improvements in TDOA geolocation.

Future studies may try to answer how to decrease the number of receivers while maintaining accuracy. Because synchronization of the receivers plays a very important role in multiple receiver systems, as in this thesis, future studies may focus on how develop better synchronization in terms of system design and used techniques.

A new design of the overall system may be another area of research, where the TDOA system and the receivers are connected to a central control

point over the Internet or over a tactical network. This may help decision makers to develop better battlefield awareness and more tactical options.

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APPENDIX A

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION TECHNIQUE

```matlab
function TDOA

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

%number of the positions of the UAV
m = 1;

k=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%Stationary Receivers
```

```matlab
X3 =-1000;      Y3 = 2000;     Z3 = -2000;
X4 = -1000;     Y4 = -1000;    Z4 = 1000;
X5 = -3000;     Y5 = -1000;    Z5 = -1000;

for i = 1:n,

    %initial coordinate matrix
    Coor2 = [0 0 0];

    %Coordinates of Receivers
    for j = 1:m,
        X1 = 1000*j;   Y1 = -3000*j;   Z1 = 1000; %Moving
UAVs
        X2 = 2000*j;   Y2 = -2000*j;   Z2 = -4000*j;

        %UAV Coordinate MAtrix
        X1Coor(1,j) = X1;
        Y1Coor(1,j) = Y1;
        Z1Coor(1,j) = Z1;

        X2Coor(1,j) = X2;
        Y2Coor(1,j) = Y2;
        Z2Coor(1,j) = Z2;

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,5e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location
```

```matlab
    %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
    a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
    a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
    a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
    b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
    a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
    a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
    a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
    b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
    a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
    a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
    a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
    b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

    %Calculate the matrix A on the left-hand side.
    A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

    %Calculate the vector B on the right-hand side.
    B = double([b1; b2; b3]);

    %Solve the simultaneous equations AX = B.

    Coor1 = (A'*A)\A'*B;

    Coor2 =+ Coor1;
```

```matlab
        end

    Coor = Coor2/m;

    %Error by every Coordinate
    Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

    %Error Matrix
    Mat(i,:) = Err;

    %Total Error
    Total_Err = sqrt(Err(1)^2 + Err(2)^2 + Err(3)^2);


end

%calculate Cov.Matrix
Cov_Mat = cov(Mat);

%Eigenvalues and eigenvectors
[Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

%Find angles (from "Computing Euler angles from a rotation
matrix")
Beta = -asind(Eigen_Vectors(3,1));
Gamma =
(atan2(Eigen_Vectors(2,1)/cosd(Beta),Eigen_Vectors(1,1)/cos
d(Beta)))*(180/pi);
Alpha =
(atan2(Eigen_Vectors(3,2)/cosd(Beta),Eigen_Vectors(3,3)/cos
d(Beta)))*(180/pi);

%Std. Dev (includes k value)
Std_Dev_Data = sqrt(diag(Eigenvalues)*k)

%Total Standat Deviation
Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)

%Draw the error ellipsoid from the diagonalized covariance
matrix
hold on
grid on
```

```matlab
%ellipsoid
[x, y, z] =
ellipsoid(E(1),E(2),E(3),Std_Dev_Data(1),Std_Dev_Data(2),St
d_Dev_Data(3),20);

%color of the ellisoid
colormap copper;

%ploy ellipsoid
hMesh = mesh(x,y,z);

%rotate ellipsoid
rotate(hMesh,[1 0 0],Alpha);
rotate(hMesh,[0 1 0],Beta);
rotate(hMesh,[0 0 1],Gamma);

%equal axis
axis equal

%# Change the camera viewpoint
view([-36 18]);

%label axises and title
xlabel('X-Axis')
ylabel('Y-Axis')
zlabel('Z-Axis')
title('TDOA Geometry and Confidence Ellipsoid')

%plot stationary receivers

plot3 (X3,Y3,Z3,'o');
plot3 (X4,Y4,Z4,'o');
plot3 (X5,Y5,Z5,'o');

%plot moving receiver
plot3(X1Coor,Y1Coor,Z1Coor,'ro');
plot3(X2Coor,Y2Coor,Z2Coor,'go');

%plot emitter
plot3 (E(1),E(2),E(3),'ms');
```

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APPENDIX B

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION
TECHNIQUE TO DECIDE THE NUMBER OF EXPERIMENTS FOR OPTIMUM
ACCURACY.

```matlab
function TDOA_Num_Experiment_vs_Accurracy

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 700;

%number of the positions of the UAV
m = 1;

kk=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%Stationary Receivers
```

```matlab
X1 = 1000;        Y1 = 1000;     Z1 = 1000;
X2 = 1000;        Y2 = 2000;     Z2 = 3000;
X3 =-1000;        Y3 = 2000;     Z3 = -2000;
X4 = -1000;       Y4 = -1000;    Z4 = 1000;
X5 = -3000;       Y5 = -1000;    Z5 = -1000;


for k = 2:n,

    for i = 2:k,

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,5e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location

        %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
        a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
        a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
        a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
        b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
        a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
```

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```matlab
        a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
        a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
        b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
        a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
        a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
        a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
        b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

        %Calculate the matrix A on the left-hand side.
        A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

        %Calculate the vector B on the right-hand side.
        B = double([b1; b2; b3]);

        %Solve the simultaneous equations AX = B.
        Coor = (A'*A)\A'*B;

        %Error by every Coordinate
        Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;

    end

    %calculate Cov.Matrix
    Cov_Mat = cov(Mat);

    %Eigenvalues and eigenvectors
    [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

    %Std. Dev (includes k value)
    Std_Dev_Data = sqrt(diag(Eigenvalues)*kk);
```

```matlab
    %Total Standat Deviation
    Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

    Tot_Std_Dev1(k,:) = Tot_Std_Dev;

end

N = linspace(1,n,n);

%plot the result
plot (N,Tot_Std_Dev1)

%label axises and title
xlabel('Number of Experiments')
ylabel('Total Standard Deviation (m)')
title('Effect of Number of Experiment to the Accuracy')
```

APPENDIX C

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION TECHNIQUE TO SEE THE EFFECTS OF DISTANCE CHANGES ON ACCURACY.

```matlab
function TDOA_Distance_to_Accuracy

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

kk=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%distance multipication constant
k=100;

%Distance Change loop
```

```matlab
for l = 1:k,

    %Distance Change in three axis
    %Stationary Receivers
    X1 = 1000*l;        Y1 = 1000*l;    Z1 = 1000*l;
    X2 = 1000*l;        Y2 = 2000*l;    Z2 = 3000*l;
    X3 =-1000*l;        Y3 = 2000*l;    Z3 = -2000*l;
    X4 = -1000*l;       Y4 = -1000*l;   Z4 = 1000*l;
    X5 = -3000*l;       Y5 = -1000*l;   Z5 = -1000*l;

    %Distance Change in two axis
    % %Stationary Receivers
    %           X1 = 1000;      Y1 = 1000*l;    Z1 = 1000*l;
    %           X2 = 1000;      Y2 = 2000*l;    Z2 = 3000*l;
    %           X3 =-1000;      Y3 = 2000*l;    Z3 = -2000*l;
    %           X4 = -1000;     Y4 = -1000*l;   Z4 = 1000*l;
    %           X5 = -3000;     Y5 = -1000*l;   Z5 = -1000*l;
    %
    %Distance Change in one axis
    %Stationary Receivers
    %           X1 = 1000;      Y1 = 1000;      Z1 = 1000*l;
    %           X2 = 1000;      Y2 = 2000;      Z2 = 3000*l;
    %           X3 =-1000;      Y3 = 2000;      Z3 = -2000*l;
    %           X4 = -1000;     Y4 = -1000;     Z4 = 1000*l;
    %           X5 = -3000;     Y5 = -1000;     Z5 = -1000*l;


    for i = 2:n,

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,5e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
```

```
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location


        %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
        a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
        a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
        a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
        b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
        a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
        a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
        a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
        b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
        a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
        a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
        a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
        b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

        %Calculate the matrix A on the left-hand side.
        A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

        %Calculate the vector B on the right-hand side.
        B = double([b1; b2; b3]);
```

```matlab
        %Solve the simultaneous equations AX = B.

        Coor = (A'*A)\A'*B;

        %Error by every Coordinate
        Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;

    end

    %calculate Cov.Matrix
    Cov_Mat = cov(Mat);

    %Eigenvalues and eigenvectors
    [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

    %Std. Dev (includes k value)
    Std_Dev_Data = sqrt(diag(Eigenvalues)*kk);

    %Total Standat Deviation
    Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

    Tot_Std_Dev1(l,:) = Tot_Std_Dev;

end

N = linspace(1,k,k);

%plot the result
plot (N,Tot_Std_Dev1)

%label axises and title
xlabel('Distance Multiplier')
ylabel('Total Standard Deviation (m)')
title('Effect of Distance to the Accuracy')
```

APPENDIX D

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION TECHNIQUE WITH STATIONARY RECEIVERS TO SEE THE EFFECTS OF NOISE ON ACCURACY.

```matlab
function TDOA_Noise_vs_Accuracy_Stationary_Receivers

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

%standard deviation of noise chage constant
l=25;

% k values for probability
kk=8.934259824; %k for 95% of probability
% kk=6.922544695; %k for 90% of probability
% kk=5.766298681; %k for 85% of probability
% kk=4.949459443; %k for 80% of probability
% kk=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;
```

```matlab
%Stationary Receivers
X1 = 1000;  Y1 = 1000;  Z1 = 1000;
X2 = 1000;  Y2 = 2000;  Z2 = 3000;
X3 =-1000;  Y3 = 2000;  Z3 = -2000;
X4 = -1000; Y4 = -1000; Z4 = 1000;
X5 = -3000; Y5 = -1000; Z5 = -1000;

%for loop for changing the standard deviation
for k = 1:l,

    %for loop for experiment for every standard deviation
    for i = 1:n,

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,k*1e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location

        %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
        a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
        a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
        a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
        b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
```

```matlab
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
        a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
        a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
        a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
        b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
        a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
        a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
        a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
        b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

        %Calculate the matrix A on the left-hand side.
        A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

        %Calculate the vector B on the right-hand side.
        B = double([b1; b2; b3]);

        %Solve the simultaneous equations AX = B.

        Coor = (A'*A)\A'*B;

        %Error by every Coordinate
        Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;

    end %i

    %calculate Cov.Matrix
    Cov_Mat = cov(Mat);
```

```matlab
    %Eigenvalues and eigenvectors
    [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

    %Std. Dev (includes k value)
    Std_Dev_Data = sqrt(diag(Eigenvalues)*kk);

    %Total Standat Deviation
    Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

    Tot_Std_Dev1(k,:) = Tot_Std_Dev;

end %k

N = linspace(1,l,l);

%plot the result
plot (N,Tot_Std_Dev1)

%label axises and title
xlabel('Noise Standard Deviation (sec)')
ylabel('Total Standard Deviation (m)')
title('Effect of Noise to the Accuracy')
```

APPENDIX E

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION TECHNIQUE WITH ONE MOVING UAV TO SEE THE EFFECTS OF NOISE ON ACCURACY.

```matlab
function TDOA_Noise_vs_Accuracy_With_Flying_UAV

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

Mat = zeros(n,3);

%number of the positions of the UAV
m = 10;

%standard deviation of noise chage constant
l=25;

% k values for probability
kk=8.934259824; %k for 95% of probability
% kk=6.922544695; %k for 90% of probability
% kk=5.766298681; %k for 85% of probability
% kk=4.949459443; %k for 80% of probability
% kk=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
```

```matlab
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%Stationary Receivers
X2 = 1000;  Y2 = 2000;  Z2 = 3000;
X3 =-1000;  Y3 = 2000;  Z3 = -2000;
X4 = -1000; Y4 = -1000; Z4 = 1000;
X5 = -3000; Y5 = -1000; Z5 = -1000;

%for loop for changing the standard deviation
for k = 1:l,

    %for loop for experiment for every standard deviation
    for i = 1:n,

        %initial coordinate matrix
        Coor2 = [0 0 0];

        %for loop for the positions of the UAV
        for j = 1:m,

            X1 = 1000*j;   Y1 = 1000*j;       Z1 = 1000;
%Moving UAV

            %Calculate the Distance From emitter to the
Receiver for TOA
            EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-
Z1)^2));
            EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-
Z2)^2));
            EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-
Z3)^2));
            EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-
Z4)^2));
            EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-
Z5)^2));

            %Generate random noise for every channel
related to Standat Deviation
            Noise_Err = random('Normal',0,k*1e-8,1,5);

            %Calculate the times of arrival.
            t1 = EC1 / (alpha1 * c) + Noise_Err(1);
            t2 = EC2 / (alpha2 * c) + Noise_Err(2);
```

```
            t3 = EC3 / (alpha3 * c) + Noise_Err(3);
            t4 = EC4 / (alpha4 * c) + Noise_Err(4);
            t5 = EC5 / (alpha5 * c) + Noise_Err(5);


            %from now on calculate the emitter location


            %Calculate the coefficients in Eqs.(17). Note
that alpha (assumed),not alpha actual is used.
            a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
            a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
            a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
            b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
            a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
            a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
            a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
            b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
            a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
            a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
            a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
            b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

            %Calculate the matrix A on the left-hand side.
            A = double([[a11 a12 a13]; [a21 a22 a23]; [a31
a32 a33]]);

            %Calculate the vector B on the right-hand side.
```

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```matlab
            B = double([b1; b2; b3]);

            %Solve the simultaneous equations AX = B.

            Coor1 = (A'*A)\A'*B;

            Coor2 =+ Coor1;

        end %j

        Coor = Coor2/m;

        %Error by every Coordinate
        Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;


    end %i

    %calculate Cov.Matrix
    Cov_Mat = cov(Mat);

    %Eigenvalues and eigenvectors
    [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);


    %Std. Dev (includes k value)
    Std_Dev_Data = sqrt(diag(Eigenvalues)*kk);

    %Total Standat Deviation
    Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

    Tot_Std_Dev1(k,:) = Tot_Std_Dev;

end %k

N = linspace(1,l,l);

%plot the result
plot (N,Tot_Std_Dev1)
```

```matlab
%label axises and title
xlabel('Noise Standard Deviation (sec))')
ylabel('Total Standard Deviation (m)')
title('Effect of Noise to the Accuracy')
```

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APPENDIX F

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION
TECHNIQUE TO SEE THE EFFECTS OF HAVING A MOVING UAV ALONG
WITH STAIONARY RECEIVERS ON ACCURACY.

```matlab
function TDOA_Flying_UAV

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, % 2006
% Latest revision: August 26, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

Mat = zeros(n,3);

%number of the positions of the UAV
m = 20;

k=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;
```

```matlab
%Stationary Receivers
X2 = 1000;        Y2 = -2000;       Z2 = -2100;
X3 =-1000;        Y3 = 2000;        Z3 = -2000;
X4 = -1000;       Y4 = -1000;       Z4 = 1000;
X5 = -3000;       Y5 = -1000;       Z5 = -1000;

%for loop for UAV movement
for j = 1:m,

    %locations of the UAV
    X1 = 1000*j;   Y1 = 1500*j;   Z1 = 2000; %Moving UAV


    %UAV Coordinate MAtrix
    X1Coor(1,j) = X1;
    Y1Coor(1,j) = Y1;
    Z1Coor(1,j) = Z1;

    %for loop for experiments for every location of the UAV
    for i = 1:n,

        %initial coordinate matrix
        Coor2 = [0 0 0];

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,5e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location
```

```matlab
        %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
        a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
        a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
        a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
        b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
        a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
        a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
        a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
        b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
        a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
        a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
        a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
        b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

        %Calculate the matrix A on the left-hand side.
        A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

        %Calculate the vector B on the right-hand side.
        B = double([b1; b2; b3]);

        %Solve the simultaneous equations AX = B.

        Coor1 = (A'*A)\A'*B;

        %Error by every Coordinate
```

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```matlab
        Err = [abs(Coor1(1)-E(1)),abs(Coor1(2)-E(2)),
abs(Coor1(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;

    end

        %calculate Cov.Matrix
        Cov_Mat = cov(Mat);

        %Eigenvalues and eigenvectors
        [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

        %Find Diagonalized Cov.Mat.
        Diagonal_Cov_Mat =
Eigen_Vectors.'*inv(Cov_Mat)*Eigen_Vectors;

        %Std. Dev (includes k value)
        Std_Dev_Data = sqrt(diag(Eigenvalues)*k);

        Std_Dev_Data_Mat(:,j) = Std_Dev_Data

        %Total Standat Deviation
        Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

        Tot_Std_Dev_Mat (j,:) = Tot_Std_Dev

end

figure

%Draw the error ellipsoid from the diagonalized covariance
matrix
hold on
grid on

%equal axis
axis equal

%# Change the camera viewpoint
view([-36 18]);

%label axises and title
```

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```matlab
xlabel('X-Axis')
ylabel('Y-Axis')
zlabel('Z-Axis')
title('TDOA Geometry')

%plot stationary receivers

plot3 (X2,Y2,Z2,'o');
plot3 (X3,Y3,Z3,'o');
plot3 (X4,Y4,Z4,'o');
plot3 (X5,Y5,Z5,'o');

%plot emitter
plot3 (E(1),E(2),E(3),'ms');

%plot moving receiver
plot3(X1Coor,Y1Coor,Z1Coor,'r--');

figure

N = linspace(1,m,m);

%plot the result
plot (N,Tot_Std_Dev_Mat)

%label axises and title
xlabel('Distance Multiplier')
ylabel('Total Standard Deviation (m)')
title('Effect of Distance to the Accuracy With UAV')
```

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MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION TECHNIQUE TO SEE THE EFFECTS OF HAVING TWO MOVING UAVS ALONG WITH STAIONARY RECEIVERS ON ACCURACY.

```matlab
function TDOA_Flying_two_UAV

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%    Closed-Form Solution of Hyperbolic Geolocation
%    Equations, 2006
% Latest revision: August 26, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

%number of the positions of the UAV
m = 20;

k=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%Stationary Receivers
X3 =-1000;      Y3 = 2000;      Z3 = -2000;
X4 = -1000;     Y4 = -1000;     Z4 = 1000;
X5 = -3000;     Y5 = -1000;     Z5 = -1000;
```

```matlab
%for loop for UAV movement
for j = 1:m,

    %locations of the UAV
    X1 = 800*j;    Y1 = 1200*j;    Z1 = 2000; %Moving UAV
    X2 = 700*j;    Y2 = -1800*j;    Z2 = 2000;

    %UAV Coordinate MAtrix
    X1Coor(1,j) = X1;
    Y1Coor(1,j) = Y1;
    Z1Coor(1,j) = Z1;

    X2Coor(1,j) = X2;
    Y2Coor(1,j) = Y2;
    Z2Coor(1,j) = Z2;

    %for loop for experiments for each location of the UAV
    for i = 1:n,

        %Calculate the Distance From emitter to the
Receiver for TOA
        EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
        EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
        EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
        EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
        EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

        %Generate random noise for every channel related to
Standat Deviation
        Noise_Err = random('Normal',0,5e-8,1,5);

        %Calculate the times of arrival.
        t1 = EC1 / (alpha1 * c) + Noise_Err(1);
        t2 = EC2 / (alpha2 * c) + Noise_Err(2);
        t3 = EC3 / (alpha3 * c) + Noise_Err(3);
        t4 = EC4 / (alpha4 * c) + Noise_Err(4);
        t5 = EC5 / (alpha5 * c) + Noise_Err(5);


        %from now on calculate the emitter location

        %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
        a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t3-t1) *(X3/alpha3^2 - X1/alpha1^2);
```

```
        a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t3-t1) *(Y3/alpha3^2 - Y1/alpha1^2);
        a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t3-t1) *(Z3/alpha3^2 - Z1/alpha1^2);
        b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
        a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t4-t1) *(X4/alpha4^2 - X1/alpha1^2);
        a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t4-t1) *(Y4/alpha4^2 - Y1/alpha1^2);
        a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t4-t1) *(Z4/alpha4^2 - Z1/alpha1^2);
        b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
        a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) -
2/(t5-t1) *(X5/alpha5^2 - X1/alpha1^2);
        a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) -
2/(t5-t1) *(Y5/alpha5^2 - Y1/alpha1^2);
        a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) -
2/(t5-t1) *(Z5/alpha5^2 - Z1/alpha1^2);
        b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

        %Calculate the matrix A on the left-hand side.
        A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);

        %Calculate the vector B on the right-hand side.
        B = double([b1; b2; b3]);

        %Solve the simultaneous equations AX = B.

        Coor1 = (A'*A)\A'*B;

        %Error by every Coordinate
        Err = [abs(Coor1(1)-E(1)),abs(Coor1(2)-E(2)),
abs(Coor1(3)-E(3))];

        %Error Matrix
        Mat(i,:) = Err;
```

```matlab
        end

        %calculate Cov.Matrix
        Cov_Mat = cov(Mat);

        %Eigenvalues and eigenvectors
        [Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

        %Find Diagonalized Cov.Mat.
        Diagonal_Cov_Mat =
Eigen_Vectors.'*inv(Cov_Mat)*Eigen_Vectors;

        %Std. Dev (includes k value)
        Std_Dev_Data = sqrt(diag(Eigenvalues)*k);

        Std_Dev_Data_Mat(:,j) = Std_Dev_Data;

        %Total Standat Deviation
        Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)
;

        Tot_Std_Dev_Mat (j,:) = Tot_Std_Dev

end

figure

%Draw the error ellipsoid from the diagonalized covariance
matrix
hold on
grid on

%equal axis
axis equal

%# Change the camera viewpoint
view([-36 18]);

%label axises and title
xlabel('X-Axis')
ylabel('Y-Axis')
zlabel('Z-Axis')
title('TDOA Geometry')
```

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```matlab
%plot stationary receivers

plot3 (X3,Y3,Z3,'o');
plot3 (X4,Y4,Z4,'o');
plot3 (X5,Y5,Z5,'o');

%plot emitter
plot3 (E(1),E(2),E(3),'ms');

%plot moving receiver
plot3(X1Coor,Y1Coor,Z1Coor,'r--');
plot3(X2Coor,Y2Coor,Z2Coor,'g--');

figure

N = linspace(1,m,m);

%plot the result
plot (N,Tot_Std_Dev_Mat)

%label axises and title
xlabel('Distance Multiplier')
ylabel('Total Standard Deviation (m)')
title('Effect of Distance to the Accuracy With UAV')
```

APPENDIX G

MATLAB CODE FOR SIMULATION OF CLOSED-FORM GEOLOCATION
TECHNIQUE TO SEE THE EFFECTS OF THE RECEIVERS GEOMETRY ON
ACCURACY.

```matlab
function TDOA_Receiver_Dist

% Written by: Volkan TAS
% Based on code developed by EZZAT G. BAKHOUM, described in
%     Closed-Form Solution of Hyperbolic Geolocation
%     Equations, % 2006
% Latest revision: August 15, 2012

%clear the screen
clc

%Variables

%number of experiments for accuracy
n = 500;

Mat = zeros(n,3);

k=8.934259824; %k for 95% of probability
% k=6.922544695; %k for 90% of probability
% k=5.766298681; %k for 85% of probability
% k=4.949459443; %k for 80% of probability
% k=4.314831079; %k for 75% of probability

%Coordinates of emitter
E=[0 0 0];

%Speed of Light(Propagation)
c = 3*10^8;

%Set All Alphas to 1
alpha1 = 1;
alpha2 = 1;
alpha3 = 1;
alpha4 = 1;
alpha5 = 1;

%Stationary Receivers
%set default for switch
```

```matlab
val = 1;

tmp = input (sprintf ('How you want to Distribute the
Receivers \n  Around the Emitter (Without Altitude
Difference)Press 1 \n  On Staright Line (Without Altitude
Difference)Press 2 \n  Around the Emitter (With Altitude
Difference) Press 3 \n  On Staright Line (With Altitude
Difference)Press 4 \n  Trapezoidal Press 5 \n
Parallelogram Press 6 \n  Inverted Triangle Press 8
','val'));

%# if the user hits 'return' without writing anything, tmp
is empty and the default is used
if ~isempty(tmp)
    val = tmp;
end

switch val

    case 1 %circle, no elevation difference
        X1 = 2e3;        Y1 = 0;           Z1 = 1.97e3;
        X2 = 7e2;        Y2 = 2e3;         Z2 = 1.98e3;
        X3 =-2e3;        Y3 = 1.5e3;       Z3 = 1.96e3;
        X4 = -2e3;       Y4 = -1.5e3;      Z4 = 2e3;
        X5 = 1e3;        Y5 = -1.5e3;      Z5 = 1.99e3;

    case 2 %On Staright Line (Without Altitude Difference)
        X1 = -2e3;       Y1 = -1e3;        Z1 = 0.99e3;
        X2 = -1e3;       Y2 = 0;           Z2 = 1e3;
        X3 = 0;          Y3 = 2e3;         Z3 = 0.98e3;
        X4 = 1e3;        Y4 = 3e3;         Z4 = 0.97e3;
        X5 = 1.5e3;      Y5 = 4e3;         Z5 = 1.1e3;

    case 3 %Around the Emitter (With Altitude Difference)
        X1 = 2e3;        Y1 = 0;           Z1 = 0;
        X2 = 0.7e3;      Y2 = 2e3;         Z2 = 1e3;
        X3 =-2e3;        Y3 = 1.5e3;       Z3 = -1e3;
        X4 = -2e3;       Y4 = -1.5e3;      Z4 = 2e3;
        X5 = 1e3;        Y5 = -1.5e3;      Z5 = -2e3;

    case 4 %On Staright Line (With Altitude Difference)
        X1 = -2e3;       Y1 = -1e3;        Z1 = 0;
        X2 = -1e3;       Y2 = 0;           Z2 = 1e3 ;
        X3 = 0;          Y3 = 2e3;         Z3 = 2e3;
        X4 = 1e3;        Y4 = 3e3;         Z4 = 3e3;
        X5 = 1.5e3;      Y5 = 4e3;         Z5 = 4e3;
```

```matlab
    case 5 %Trapezoidal
        X1 = 1e3;          Y1 = 0;           Z1 = 0;
        X2 = 2e3;          Y2 = 2e3;         Z2 = -0.1e3;
        X3 = 3e3;          Y3 = 2e3;         Z3 = -0.1e3;
        X4 = 4e3;          Y4 = 2e3;         Z4 = -0.1e3;
        X5 = 5e3;          Y5 = 0;           Z5 = -0.1e3;

    case 6 %Parallelogram
        X1 = -1e3;         Y1 = 0;           Z1 = 0;
        X2 = -2e3;         Y2 = 2e3;         Z2 = -0.1e3;
        X3 = 0;            Y3 = 2e3;         Z3 = -0.1e3;
        X4 = -2e3;         Y4 = 0;           Z4 = -0.1e3;
        X5 = -3e3;         Y5 = 0;           Z5 = -0.1e3;

    case 7 %Lorenze
        X1 = 8e3;          Y1 = 15e3;        Z1 = -0.12e3;
        X2 = -13e3;        Y2 = 12.5e3;      Z2 = -0.13e3;
        X3 = 0;            Y3 = 18e3;        Z3 = -0.14e3;
        X4 = 13e3;         Y4 = 12.5e3;      Z4 = -0.11e3;
        X5 = 0;            Y5 = 8;           Z5 = -0.1e3;

    case 8 %Inverted Triangle
        X1 = 2e3;          Y1 = 0;           Z1 = 0;
        X2 = 4e3;          Y2 = 2e3;         Z2 = -0.1e3;
        X3 = 6e3;          Y3 = -2e3;        Z3 = -0.1e3;
        X4 = 8e3;          Y4 = 4e3;         Z4 = -0.1e3;
        X5 = 10e3;         Y5 = -4e3;        Z5 = -0.1e3;
end

for i = 1:n,

    %Calculate the Distance From emitter to the Receiver
for TOA
    EC1=sqrt(abs((E(1)-X1)^2+(E(2)-Y1)^2+(E(3)-Z1)^2));
    EC2=sqrt(abs((E(1)-X2)^2+(E(2)-Y2)^2+(E(3)-Z2)^2));
    EC3=sqrt(abs((E(1)-X3)^2+(E(2)-Y3)^2+(E(3)-Z3)^2));
    EC4=sqrt(abs((E(1)-X4)^2+(E(2)-Y4)^2+(E(3)-Z4)^2));
    EC5=sqrt(abs((E(1)-X5)^2+(E(2)-Y5)^2+(E(3)-Z5)^2));

    %Generate random noise for every channel related to
Standat Deviation
    Noise_Err = random('Normal',0,5e-8,1,5);

    %Calculate the times of arrival.
    t1 = EC1 / (alpha1 * c) + Noise_Err(1);
```

```
    t2 = EC2 / (alpha2 * c) + Noise_Err(2);
    t3 = EC3 / (alpha3 * c) + Noise_Err(3);
    t4 = EC4 / (alpha4 * c) + Noise_Err(4);
    t5 = EC5 / (alpha5 * c) + Noise_Err(5);


    %from now on calculate the emitter location


    %Calculate the coefficients in Eqs.(17). Note that
alpha (assumed),not alpha actual is used.
    a11 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) - 2/(t3-
t1) *(X3/alpha3^2 - X1/alpha1^2);
    a12 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) - 2/(t3-
t1) *(Y3/alpha3^2 - Y1/alpha1^2);
    a13 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) - 2/(t3-
t1) *(Z3/alpha3^2 - Z1/alpha1^2);
    b1 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t3-t1) *
((X3^2+Y3^2+Z3^2)/alpha3^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t3-t2);
    a21 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) - 2/(t4-
t1) *(X4/alpha4^2 - X1/alpha1^2);
    a22 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) - 2/(t4-
t1) *(Y4/alpha4^2 - Y1/alpha1^2);
    a23 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) - 2/(t4-
t1) *(Z4/alpha4^2 - Z1/alpha1^2);
    b2 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t4-t1) *
((X4^2+Y4^2+Z4^2)/alpha4^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t4-t2);
    a31 = 2/(t2-t1) * (X2/alpha2^2 - X1/alpha1^2) - 2/(t5-
t1) *(X5/alpha5^2 - X1/alpha1^2);
    a32 = 2/(t2-t1) * (Y2/alpha2^2 - Y1/alpha1^2) - 2/(t5-
t1) *(Y5/alpha5^2 - Y1/alpha1^2);
    a33 = 2/(t2-t1) * (Z2/alpha2^2 - Z1/alpha1^2) - 2/(t5-
t1) *(Z5/alpha5^2 - Z1/alpha1^2);
    b3 = 1/(t2-t1) * ((X2^2+Y2^2+Z2^2)/alpha2^2-
(X1^2+Y1^2+Z1^2)/alpha1^2)- 1/(t5-t1) *
((X5^2+Y5^2+Z5^2)/alpha5^2- (X1^2+Y1^2+Z1^2)/alpha1^2) +
c^2 * (t5-t2);

    %Calculate the matrix A on the left-hand side.
    A = double([[a11 a12 a13]; [a21 a22 a23]; [a31 a32
a33]]);
```

```matlab
    %Calculate the vector B on the right-hand side.
    B = double([b1; b2; b3]);

    %Solve the simultaneous equations AX = B.

    Coor = (A'*A)\A'*B;


    %Error by every Coordinate
    Err = [abs(Coor(1)-E(1)),abs(Coor(2)-E(2)),
abs(Coor(3)-E(3))];

    %Error Matrix
    Mat(i,:) = Err;


end

%calculate Cov.Matrix
Cov_Mat = cov(Mat);

%Eigenvalues and eigenvectors
[Eigen_Vectors,Eigenvalues] = eig(Cov_Mat);

%Find angles (from "Computing Euler angles from a rotation
matrix")
Beta = -asind(Eigen_Vectors(3,1));
Gamma =
(atan2(Eigen_Vectors(2,1)/cosd(Beta),Eigen_Vectors(1,1)/cos
d(Beta)))*(180/pi);
Alpha =
(atan2(Eigen_Vectors(3,2)/cosd(Beta),Eigen_Vectors(3,3)/cos
d(Beta)))*(180/pi);


%Std. Dev (includes k value)
Std_Dev_Data = sqrt(diag(Eigenvalues)*k)

%Total Standat Deviation
Tot_Std_Dev =
sqrt(Std_Dev_Data(1)^2+Std_Dev_Data(2)^2+Std_Dev_Data(3)^2)

%Draw the error ellipsoid from the diagonalized covariance
matrix
hold on
grid on
```

```matlab
%ellipsoid
[x, y, z] =
ellipsoid(E(1),E(2),E(3),Std_Dev_Data(1),Std_Dev_Data(2),St
d_Dev_Data(3),20);

%color of the ellisoid
colormap copper;

%ploy ellipsoid
hMesh = mesh(x,y,z);

%rotate ellipsoid
rotate(hMesh,[1 0 0],Alpha);
rotate(hMesh,[0 1 0],Beta);
rotate(hMesh,[0 0 1],Gamma);

%equal axis
axis equal

%# Change the camera viewpoint
view([-36 18]);

%label axises and title
xlabel('X-Axis')
ylabel('Y-Axis')
zlabel('Z-Axis')
title('TDOA Geometry and Confidence Ellipsoid')

%plot stationary receivers
plot3 (X1,Y1,Z1,'o');
plot3 (X2,Y2,Z2,'o');
plot3 (X3,Y3,Z3,'o');
plot3 (X4,Y4,Z4,'o');
plot3 (X5,Y5,Z5,'o');

%plot emitter
plot3 (E(1),E(2),E(3),'ms');
```

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